Day 341 **S Tier 10-a-day: Model Solutions**

1 Arithmetic sequences

The 6th term of an arithmetic sequence is 33.
The 11th term of the sequence is 43.
(a) Find an expression for the nth term of the sequence.
(b) Is 99 a term of the sequence? You must show your working.

(a) Common difference = (43 − 33) ÷ (11 − 6) = 2
nth term = 2n + c. For n = 6: 2 × 6 + c = 33, so c = 21
nth term = 2n + 21
(b) Solve 2n + 21 = 99: 2n = 78, so n = 39
n is a positive whole number, so 99 is the 39th term
Answer: (a) 2n + 21 (b) Yes, it is the 39th term

2 Weighted percentages

The ratio of Year 10 students to Year 11 students in a school is 1 : 4.
80% of the Year 10 students and 85% of the Year 11 students have a part-time job.
What percentage of all of these students have a part-time job?

Imagine 100 students in total: 20 Year 10 students and 80 Year 11 students
80% of 20 = 16 and 85% of 80 = 68
Together: 84 out of 100
84 ÷ 100 × 100 = 84%
Answer: 84%

3 Multiplying and dividing surds

Work out $(6\sqrt{6})^2 - (4\sqrt{6})^2$

$(6\sqrt{6})^2 = 6^2 \times 6 = 216$
$(4\sqrt{6})^2 = 4^2 \times 6 = 96$
216 − 96 = 120
Answer: 120

4 Exchange rates

Nadia changes £700 into dollars at £1 = $1.25.
Nadia spends $650 and then changes the rest back into pounds at £1 = $1.5.
How many pounds does Nadia get back?

£700 × 1.25 = $875
875 − 650 = $225
225 ÷ 1.5 = £150
Answer: £150

5 Vectors in a triangle

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 3 : 2. N is on AB with AN : NB = 1 : 3.
(a) Find ON in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Find MN in terms of $\mathbf{a}$ and $\mathbf{b}$.
The diagram is not drawn accurately.

(a) AB = $-\mathbf{a} + \mathbf{b}$, so AN = $\frac{1}{4}(\mathbf{b} - \mathbf{a})$

ON = OA + AN = $\mathbf{a} + \frac{1}{4}(\mathbf{b} - \mathbf{a}) = \frac{3}{4}\mathbf{a} + \frac{1}{4}\mathbf{b}$

(b) OM = $\frac{3}{5}\mathbf{a}$

MN = ON − OM = $\frac{3}{4}\mathbf{a} + \frac{1}{4}\mathbf{b} - \frac{3}{5}\mathbf{a} = \frac{3}{20}\mathbf{a} + \frac{1}{4}\mathbf{b}$

Answer: (a) $\frac{3}{4}\mathbf{a} + \frac{1}{4}\mathbf{b}$ (b) $\frac{3}{20}\mathbf{a} + \frac{1}{4}\mathbf{b}$

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6 Standard form problems

A library has 2×10^3 books.
Each book has 4.9×10^3 words.
Work out the total number of words in all the books.
Give your answer in standard form.

Total = $(2 \times 10^3) \times (4.9 \times 10^3)$
$2 \times 4.9 = 9.8$ and $10^3 \times 10^3 = 10^6$
$= 9.8 \times 10^6$
Answer: 9.8×10^6 words

7 Line of best fit

The scatter graph shows temperature (x) and ice creams sold (y).
The line of best fit passes through (4, 28) and (8, 44).
(a) Find its equation in the form y = mx + c.
(b) Explain what its gradient tells you.

(a) Gradient m = $\frac{44 - 28}{8 - 4} = \frac{16}{4} = 4$
y = 4x + c: using (4, 28), 28 = 4 × 4 + c, so c = 12
y = 4x + 12
(b) For each extra degree of temperature, the number of ice creams sold goes up by 4
Answer: (a) y = 4x + 12 (b) For each extra degree of temperature, the number of ice creams sold goes up by 4

8 Circle and line intersections

Solve the simultaneous equations
$x^2 + y^2 = 20$
$y = -3x - 10$

Substitute into the first equation: $x^2 + (-3x - 10)^2 = 20$
Expand: $10x^2 + 60x + 80 = 0$
Divide by 10: $x^2 + 6x + 8 = 0$
Factorise: $(x + 4)(x + 2) = 0$, so $x = -4$ or $x = -2$
$y = -3x - 10$ gives $y = 2$ and $y = -4$
Answer: $x = -4$, $y = 2$ and $x = -2$, $y = -4$

9 Bounds of roots

The area of a square is 186 cm^2, correct to the nearest cm^2.
Work out the lower bound of the side length of the square.
Give your answer correct to 3 significant figures. (Calculator)

$185.5 \leq$ area < 186.5
Lower bound of side = $\sqrt{185.5}$
= 13.6198... = 13.6 (3 s.f.)
Answer: 13.6 cm

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10 Area of a triangle using sine

Triangle ABC has AB = 9 cm and BC = 4 cm.
The area of triangle ABC is 9 cm^2.
Angle ABC is acute.
Work out the size of angle ABC.
Do not use a calculator.

Area = $\frac{1}{2} \times AB \times BC \times \sin B$

$9 = \frac{1}{2} \times 9 \times 4 \times \sin B$

$\sin B = 9 \div 18 = \frac{1}{2}$

$\sin 30° = \frac{1}{2}$, and B is acute, so B = 30°

Answer: 30°