Day 340 **S Tier 10-a-day: Model Solutions**

1 Expanding brackets with surds

Expand and simplify $(1 + 3\sqrt{6})(1 - \sqrt{6})$

Give your answer in the form $p + q\sqrt{6}$

$(1 + 3\sqrt{6})(1 - \sqrt{6}) = 1 - \sqrt{6} + 3\sqrt{6} - 18$

$= -17 + 2\sqrt{6}$

Answer: $-17 + 2\sqrt{6}$

2 Similar shapes

Two shapes are mathematically similar.

The smaller has height 6 cm and area 12 cm^2.

The larger has area 75 cm^2.

Work out the height of the larger shape.

Ratio of areas = 75 : 12 = 25 : 4

Ratio of lengths = 5 : 2

Height = $6 \times \frac{5}{2} = 15$ cm

Answer: 15 cm

3 Factorising quadratics

Factorise $3x^2 + 14x + 15$

$3 \times 15 = 45$. Two numbers that multiply to 45 and add to 14 are 9 and 5

Split the middle term: $3x^2 + 9x + 5x + 15$

Factorise in pairs: $3x(x + 3) + 5(x + 3)$

$= (3x + 5)(x + 3)$

Answer: $(3x + 5)(x + 3)$

4 Fractions of fractions

Ravi cycles $\frac{4}{5}$ of a route before lunch.

After lunch he cycles $\frac{2}{7}$ of the remaining distance.

He still has 15 km to go.

How long is the route, in km?

After the first part, $\frac{1}{5}$ remains

After the second, $\frac{5}{7}$ of $\frac{1}{5}$ remains = $\frac{1}{7}$ of the total

$\frac{1}{7}$ of the total is 15

Total = $15 \div \frac{1}{7} = 105$

Answer: 105 km

5 Forming and solving quadratic equations

The product of two positive numbers is 6.

The larger number is 1 more than the smaller number.

Let the smaller number be n.

(a) Show that $n^2 + n - 6 = 0$

(b) Work out the two numbers.

(a) The numbers are n and n + 1, so $n(n + 1) = 6$

$n^2 + n = 6$, so $n^2 + n - 6 = 0$

(b) Factorise: $(n - 2)(n + 3) = 0$

$n = 2$ or $n = -3$. n is positive, so $n = 2$

The numbers are 2 and 3

Answer: (b) 2 and 3

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6 Using a line of best fit

The scatter graph shows age of car (x) and value (y).
The equation of the line of best fit is $y = -2x + 25$.
(a) Estimate y when x = 9.
(b) Explain why an estimate of y when x = 20 may be unreliable.

(a) $y = -2 \times 9 + 25 = 7$
(b) x = 20 is outside the range of the data on the graph, so the estimate relies on the pattern continuing (extrapolation)
Answer: (a) 7 (b) x = 20 is outside the range of the data

7 Angles of elevation

A point A is on level ground in line with the base of a lighthouse. (Calculator)
The angle of elevation of the top of the lighthouse from A is 36°.
B is a point on the same line, 25 m further from the lighthouse than A.
The angle of elevation of the top of the lighthouse from B is 19°.
Work out the height of the lighthouse. Give your answer to 1 decimal place.

Let the height be h and the distance from A to the base be x. $\tan 36° = \frac{h}{x}$, so $x = h \div \tan 36°$

From B: $\tan 19° = \frac{h}{x + 25}$, so $x + 25 = h \div \tan 19°$

Subtract: $25 = h \div \tan 19° - h \div \tan 36° = h(2.9042 - 1.3764)$
$h = 25 \div 1.5278 = 16.4$ m (1 d.p.)
Answer: 16.4 m

8 Perpendicular bisector

A is the point (−1, −2) and B is the point (9, 0).
Find an equation of the perpendicular bisector of AB.
Give your answer in the form ax + by = c, where a, b and c are integers.

Midpoint of AB = $(\frac{-1 + 9}{2}, \frac{-2 + 0}{2}) = (4, -1)$

Gradient of AB = $\frac{0 - (-2)}{9 - (-1)} = \frac{1}{5}$

The perpendicular gradient is $-1 \div \frac{1}{5} = -5$

Using the midpoint: $y + 1 = -5(x - 4)$
Rearrange: $5x + y = 19$
Answer: 5x + y = 19

9 Combining means

The mean of the marks in a test for the boys is 79.
The mean of the marks in a test for the girls is 61.
The mean for all of the boys and girls together is 71.
Work out the ratio of the number of boys to the number of girls.

Let there be p boys and q girls
Total = 79p + 61q, and the overall mean gives $79p + 61q = 71(p + q)$
$8p = 10q$
$p : q = 10 : 8 = 5 : 4$
Answer: 5 : 4

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10 Enlargement

Triangle P has vertices A(−1, 4), B(3, 0) and C(−7, 6).
Triangle Q has vertices A'(−1, 1), B'(−3, 3) and C'(2, 0).
Q is an enlargement of P.
Describe fully the single transformation that maps P onto Q.

x-step from A to B = 4, x-step from A' to B' = −2

Scale factor = $-2 \div 4 = -\frac{1}{2}$

The centre C satisfies: $C + -\frac{1}{2}(A - C) = A'$, so $(1 - -\frac{1}{2})C = A' - -\frac{1}{2}A$

x: $\frac{3}{2} \times x = (-1) - (-\frac{1}{2}) \times (-1) = -\frac{3}{2}$, so $x = -1$

y: $\frac{3}{2} \times y = 1 - (-\frac{1}{2}) \times 4 = 3$, so $y = 2$

Answer: Enlargement, scale factor $-\frac{1}{2}$, centre $(-1, 2)$