Day 34 **S Tier 10-a-day: Model Solutions**

1 Line and curve intersections

Solve the simultaneous equations

$y = x^2 + 4x + 3$

$y = 5x + 23$

Substitute: $x^2 + 4x + 3 = 5x + 23$

Rearrange: $x^2 - x - 20 = 0$

Factorise: $(x + 4)(x - 5) = 0$, so $x = -4$ or $x = 5$

When $x = -4$: $y = 5 \times (-4) + 23 = 3$

When $x = 5$: $y = 5 \times 5 + 23 = 48$

Answer: $x = -4$, $y = 3$ and $x = 5$, $y = 48$

2 Fractions removed from a ratio

The ratio of men to women people at a meeting is 5 : 4.

Then $\frac{1}{5}$ of the men and $\frac{1}{6}$ of the women are taken out.

Work out the ratio of men to women now.

Left: $\frac{4}{5} \times 5 = 4$ of the men part, $\frac{5}{6} \times 4 = \frac{10}{3}$ of the women part

$4 : \frac{10}{3}$

Multiply by 3: 12 : 10 = 6 : 5

Answer: 6 : 5

3 Enlargement

Triangle P has vertices A(1, −3), B(−2, −4) and C(1, 0).

Triangle Q has vertices A'(−1, −9), B'(−10, −12) and C'(−1, 0).

Q is an enlargement of P.

Describe fully the single transformation that maps P onto Q.

x-step from A to B = −3, x-step from A' to B' = −9

Scale factor = −9 ÷ −3 = 3

The centre C satisfies: $C + 3(A - C) = A'$, so $(1 - 3)C = A' - 3A$

x: $(-2) \times x = (-1) - 3 \times 1 = -4$, so $x = 2$

y: $(-2) \times y = (-9) - 3 \times (-3) = 0$, so $y = 0$

Answer: Enlargement, scale factor 3, centre (2, 0)

4 Percentage change in area and volume

The radius of a sphere is decreased by 40%.

Work out the percentage change in the volume of the sphere.

Radius multiplier = 0.6

Volume multiplier = $0.6^3 = 0.216$

0.216 − 1 = −0.784, so a 78.4% decrease

Answer: 78.4% decrease

5 Bounds in geometry

A rectangle has length 5.5 cm and width 6.3 cm, both correct to 1 decimal place.

Work out the lower bound of the perimeter of the rectangle.

Each length is given to 1 decimal place, so it can be up to 0.05 cm either side

Length is between 5.45 and 5.55, width is between 6.25 and 6.35

Perimeter = 2 × (5.45 + 6.25) = 23.4 cm

Answer: 23.4 cm

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6 Speed-time graphs

The graph shows the speed of a train between two stations.
(a) Work out the acceleration in the first 8 seconds.
(b) Work out the total distance.

(a) Acceleration = change in speed ÷ time = 16 ÷ 8 = 2 m/s^2
(b) Distance = area under the graph, split into two triangles and a rectangle
$\frac{1}{2} \times 8 \times 16 + 16 \times 30 + \frac{1}{2} \times 16 \times 16 = 64 + 480 + 128$
$= 672$
Answer: (a) 2 m/s^2 (b) 672 m

7 Fraction problems

A bath is $\frac{1}{4}$ full.

170 more litres of water are added and the bath is then $\frac{3}{4}$ full.

Work out the capacity of the bath.

Fraction added = $\frac{3}{4} - \frac{1}{4} = \frac{1}{2}$

$\frac{1}{2}$ of the capacity is 170 litres

Capacity = $170 \div \frac{1}{2} = 340$

Answer: 340 litres

8 Growth and decay

A rare car is worth £10,000.
Its value increases by 6% each year.
After how many whole years will it first be worth more than £12,500? (Calculator)

Multiplier each year = 1.06
After 3 years: £10,000 × 1.06^3 = £11,910.16
After 4 years: £10,000 × 1.06^4 = £12,624.77
So the first time it is more than £12,500 is after 4 years
Answer: 4 years

9 Tangent to a circle

A circle has equation $x^2 + y^2 = 625$.
The point P(7, 24) lies on the circle.
(a) Find the equation of the tangent to the circle at P. Give your answer in the form ax + by = c, where a, b and c are integers.
(b) The tangent meets the x-axis at the point T. Write down the coordinates of T.

(a) The gradient of the radius OP is $\frac{24}{7}$

The tangent is perpendicular to the radius, so its gradient is $-1 \div \frac{24}{7} = -\frac{7}{24}$

$y - 24 = (-\frac{7}{24})(x - 7)$
Multiply out and rearrange: $7x + 24y = 625$
(b) On the x-axis y = 0, so 7x = 625
$x = \frac{625}{7}$, so T is $(\frac{625}{7}, 0)$

Answer: (a) 7x + 24y = 625 (b) T = $(\frac{625}{7}, 0)$

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10 Ratio and money transferred

Sofia and Amir share some money in the ratio 2 : 1.
Sofia gives Amir £3.
The ratio of their money is now 5 : 4.
How much money do they have altogether?

Let the amounts be 2k and 1k, so the total is 3k

Sofia now has $2k - 3$, which is $\frac{5}{9}$ of the total

$9(2k - 3) = 5 \times 3k$
$18k - 27 = 15k$, so $3k = 27$ and $k = 9$
Total = $3 \times 9 = £27$
Answer: £27