Day 339 **S Tier 10-a-day: Model Solutions**

1 Reverse percentage growth and decay

The population of a town grows by 5% each year.
After 3 years the population is 11,576.
Work out the population 3 years ago. (Calculator)

Yearly multiplier = 1.05
Original × 1.05^3 = 11,576
Original = 11,576 ÷ 1.05^3 = 10,000
Answer: 10,000

2 Ordering numbers

Write these numbers in order of size, starting with the smallest.

0.35, 39%, $\frac{1}{3}$, $\frac{7}{13}$

Convert each to a decimal: 0.35 = 0.35, 39% = 0.39, $\frac{1}{3}$ = 0.333, $\frac{7}{13}$ = 0.538

In order: $\frac{1}{3}$, 0.35, 39%, $\frac{7}{13}$

Answer: $\frac{1}{3}$, 0.35, 39%, $\frac{7}{13}$

3 Compound interest

Priya invests £7,500. The rates of interest for the next 3 years are 2%, 3% and 5%.
Compound interest is paid. Work out the value of the investment after 3 years.
Give your answer to the nearest penny. (Calculator)

Multipliers: 1.02, 1.03 and 1.05
7500 × 1.02 × 1.03 × 1.05 = 8273.48
To the nearest penny: £8,273.48
Answer: £8,273.48

4 Simplifying algebraic fractions

Simplify fully $\frac{5x^2 - 180}{x^2 - 8x + 12}$

Factorise the top: $5(x - 6)(x + 6)$
Factorise the bottom: $(x - 6)(x - 2)$
Cancel the common factor $(x - 6)$: $\frac{5(x + 6)}{x - 2}$

Answer: $\frac{5(x + 6)}{x - 2}$

5 Column vectors

$\mathbf{a} = \begin{pmatrix} 6 \\ 5 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$

$\mathbf{a} + k\mathbf{b}$ is parallel to $\mathbf{c}$.
Work out the value of k.

$\mathbf{a} + k\mathbf{b} = \begin{pmatrix} 6 + 1k \\ 5 + 2k \end{pmatrix}$

Parallel to $\mathbf{c}$: $\frac{6 + 1k}{2} = \frac{5 + 2k}{2}$

$2(6 + 1k) = 2(5 + 2k)$
$k \times (-2) = -2$
$k = 1$
Answer: $\mathbf{k} = 1$

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6 Algebraic fractions: adding and subtracting

Show that $\frac{5}{3x-5} + \frac{x+3}{2x-1}$ can be written as $\frac{3x^2+14x-20}{(3x-5)(2x-1)}$

The common denominator is $(3x-5)(2x-1)$

$$\frac{5}{3x-5} + \frac{x+3}{2x-1} = \frac{5(2x-1)+(x+3)(3x-5)}{(3x-5)(2x-1)}$$

Expand the numerator: $3x^2 + 14x - 20$

$$\frac{3x^2+14x-20}{(3x-5)(2x-1)}$$

Answer: $\mathbf{\frac{3x^2+14x-20}{(3x-5)(2x-1)}}$

7 HCF and LCM

The HCF of two numbers is 5 and their LCM is 275.
One of the numbers is 25.
Work out the other number.

HCF × LCM = product of the two numbers
5 × 275 = 1375
Other number = 1375 ÷ 25 = 55
Answer: 55

8 Exact lengths and areas

A rectangle has area $(14 + 10\sqrt{2})$ cm^2 and length $(2 + 2\sqrt{2})$ cm.
Find the width of the rectangle.
Give your answer in the form $a + b\sqrt{2}$

Width = area ÷ length = $\frac{14+10\sqrt{2}}{2+2\sqrt{2}}$

Multiply top and bottom by $(2 - 2\sqrt{2})$:
Numerator: $(14 + 10\sqrt{2})(2 - 2\sqrt{2}) = -12 - 8\sqrt{2}$
Denominator: $4 - 8 = -4$
Width = $3 + 2\sqrt{2}$
Answer: $3 + 2\sqrt{2}$ cm

9 Geometric sequences

The 3rd term of a geometric sequence is 162.
The 5th term of the sequence is 1458.
All the terms of the sequence are positive.
(a) Work out the common ratio.
(b) Work out the first term.

(a) The common ratio r satisfies $r^2 = 1458 \div 162 = 9$
$r = \sqrt{9} = 3$ (positive, since all terms are positive)
(b) The 3rd term is $a \times r^2$: $a \times 9 = 162$
$a = 162 \div 9 = 18$
Answer: (a) r = 3 (b) 18

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10 Vectors: intersecting lines

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 1 : 3. N is on OB with ON : NB = 1 : 2. AN and BM intersect at X.
(a) Find BM in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Find OX in terms of $\mathbf{a}$ and $\mathbf{b}$.
The diagram is not drawn accurately.

(a) OM = $\frac{1}{4}\mathbf{a}$, so BM = $-\mathbf{b} + \frac{1}{4}\mathbf{a} = \frac{1}{4}\mathbf{a} - \mathbf{b}$

(b) X is on BM: BX = s BM, so OX = OB + s BM = $\frac{1}{4}s\mathbf{a} + (1 - s)\mathbf{b}$

X is on AN: OX = OA + t AN = OA + t(ON − OA) = $(1 - t)\mathbf{a} + \frac{1}{3}t\mathbf{b}$

Equate: $\frac{1}{4}s = 1 - t$ and $1 - s = \frac{1}{3}t$

Solving: s = $\frac{8}{11}$ and t = $\frac{9}{11}$

OX = $\frac{2}{11}\mathbf{a} + \frac{3}{11}\mathbf{b}$

Answer: (a) $\frac{1}{4}\mathbf{a} - \mathbf{b}$ (b) OX = $\frac{2}{11}\mathbf{a} + \frac{3}{11}\mathbf{b}$