Day 338 **S Tier 10-a-day: Model Solutions**

1 Factorising quadratics

Factorise $16x^2 + 8x - 3$

$16 \times (-3) = -48$. Two numbers that multiply to -48 and add to 8 are -4 and 12

Split the middle term: $16x^2 - 4x + 12x - 3$

Factorise in pairs: $4x(4x - 1) + 3(4x - 1)$

$= (4x + 3)(4x - 1)$

Answer: $(4x + 3)(4x - 1)$

2 Capture and recapture

60 fish in a lake are caught, tagged and put back.

Later, a second sample of 100 fish is caught. 8 of these are tagged.

(a) Work out an estimate for the number of fish in a lake.

(b) State one assumption you have made.

(a) The proportion tagged in the second sample is 8 out of 100

This should match the proportion tagged in the whole population: $\frac{8}{100} = \frac{60}{N}$

$N = 60 \times 100 \div 8 = 750$

(b) No fish are born, die or leave between the samples, and the fish mix evenly

Answer: (a) 750 (b) The population stays the same between samples

3 Sine rule

The diagram shows triangle ABC. (Calculator)

Work out the length of AC. Give your answer correct to 1 decimal place.

The diagram is not drawn accurately.

Use the sine rule: $\frac{AC}{\sin B} = \frac{BC}{\sin A}$

$\frac{x}{\sin 69°} = \frac{10}{\sin 61°}$

$x = \frac{10 \times \sin 69°}{\sin 61°} = 10.674$

$x = 10.7$ cm (1 d.p.)

Answer: AC = 10.7 cm

4 Error intervals

The capacity of a tank is 0.88 litres, correct to 2 significant figures.

Write down the error interval for the capacity of a tank, using c.

The value has been rounded to the nearest 0.01

Half of this is 0.005, so go 0.005 below and 0.005 above

$0.88 - 0.005 = 0.875$ and $0.88 + 0.005 = 0.885$

$0.875 \leq c < 0.885$

Answer: $0.875 \leq c < 0.885$

5 Hollow cylinder and density

A metal pipe is a hollow cylinder of length 20 cm. (Calculator)

The outer radius is 3 cm and the inner radius is 1 cm.

The density of the metal is 8.9 g/cm^3.

Work out the mass of the pipe. Give your answer in kilograms correct to 3 significant figures.

Volume of metal $= \pi \times 3^2 \times 20 - \pi \times 1^2 \times 20 = \pi(9 - 1) \times 20$

$= 502.65$ cm^3

Mass = density × volume $= 8.9 \times 502.65 = 4473.6$ g

$= 4.47$ kg (3 s.f.)

Answer: 4.47 kg

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6 Quadratic inequalities

Solve $3x^2 \geq -9x$

Rearrange so that one side is 0: $3x^2 + 9x \geq 0$
Divide by 3: $x^2 + 3x \geq 0$
Solve $x^2 + 3x = 0$: $(x + 3)(x) = 0$, so $x = -3$ or $x = 0$
The graph is a U shape crossing the x-axis at -3 and 0; it is above the axis outside the roots
$x \leq -3$ or $x \geq 0$
Answer: $x \leq -3$ or $x \geq 0$

7 Laws of indices

Write $\frac{16^{n+1} \times 8^{n+3}}{8^n}$ in the form 2^{an+c}
where a and c are integers.

$16 = 2^4$, $8 = 2^3$, $8 = 2^3$
$16^{n+1} = 2^{4(n+1)}$, $8^{n+3} = 2^{3(n+3)}$, $8^n = 2^{3n}$
Exponent: $4(n + 1) + 3(n + 3) - 3n = 4n + 13$
Answer: a = 4, c = 13

8 Rationalising the denominator

Write $\frac{6+\sqrt{2}}{2-\sqrt{2}}$ in the form $p + q\sqrt{2}$
where p and q are rational numbers.

Multiply top and bottom by $(2 + \sqrt{2})$:
Numerator: $(6 + \sqrt{2})(2 + \sqrt{2}) = 14 + 8\sqrt{2}$
Denominator: $4 - 2 = 2$
$= 7 + 4\sqrt{2}$
Answer: $7 + 4\sqrt{2}$

9 Loci and regions

ABCD is a rectangle with AB = 8 cm and AD = 11 cm.
A point P inside the rectangle is such that AP ≤ 8 cm and P is nearer to A than to B.
Work out the area of the region in which P can lie. Give your answer in terms of π and surds.

Nearer to A than to B: P is on the A side of the perpendicular bisector of AB, a distance 4 cm from A
AP ≤ 8 cm: P is inside the quarter circle of radius 8 cm centred at A

The bisector meets the arc where $\cos\theta = \frac{4}{8}$, so $\theta = 60°$ at A. The region is a right-angled triangle plus a sector of angle 30°

Triangle: base 4 cm, height $8 \sin 60° = 4\sqrt{3}$ cm, area $= \frac{1}{2} \times 4 \times 4\sqrt{3} = 8\sqrt{3}$

Sector of angle 30° : $\frac{30}{360} \times \pi \times 8^2 = \frac{16}{3}\pi$

Area $= 8\sqrt{3} + \frac{16}{3}\pi$ cm^2

Answer: $(8\sqrt{3} + \frac{16}{3}\pi)$ cm^2

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10 Ratio and money transferred

Liam and Fatima share some money in the ratio 6 : 1.
Liam gives Fatima £8.
The ratio of their money is now 5 : 2.
How much money do they have altogether?

Let the amounts be 6k and 1k, so the total is 7k

Liam now has $6k - 8$, which is $\frac{5}{7}$ of the total

$7(6k - 8) = 5 \times 7k$
$42k - 56 = 35k$, so $7k = 56$ and $k = 8$
Total $= 7 \times 8 = £56$
Answer: £56