Day 337 **S Tier 10-a-day: Model Solutions**

1 Stratified sampling

The table shows the number of students in each of four groups.
(a) A stratified sample of 27 is taken. Work out how many come from Year 10.
(b) A different stratified sample has 6 from Year 7. Work out the size of this sample.

Total = 75 + 275 + 250 + 75 = 675

(a) Year 10: $\frac{75}{675} \times 27 = 3$

(b) Fraction sampled = $6 \div 75 = \frac{2}{25}$, so sample size = $\frac{2}{25} \times 675 = 54$

Answer: (a) 3 (b) 54

2 Simplifying surds

Write $\sqrt{108} - \sqrt{12} + \sqrt{48}$ in the form $a\sqrt{3}$
where a is an integer.

$\sqrt{108} = 6\sqrt{3}$; $\sqrt{12} = 2\sqrt{3}$; $\sqrt{48} = 4\sqrt{3}$
$(6 - 2 + 4)\sqrt{3} = 8\sqrt{3}$
Answer: $8\sqrt{3}$

3 Mean of a changing set

The mean of 11 numbers is 53.
A new number is added and the mean of all 12 numbers is 49.75.
Work out the new number.

Total of the 11 numbers = 11 × 53 = 583
New total = 12 × 49.75 = 597
New number = 597 − 583 = 14
Answer: 14

4 Rearranging when the subject appears twice

Make u the subject of $v = \frac{3u + 2}{u - 4}$

Multiply by u − 4: v(u − 4) = 3u + 2
Expand: uv − 4v = 3u + 2
Collect the u terms: uv − 3u = 4v + 2
Factorise: u(v − 3) = 4v + 2
$u = \frac{4v + 2}{v - 3}$

Answer: $\mathbf{u = \frac{4v + 2}{v - 3}}$

5 Areas of similar triangles

In the diagram, DE is parallel to BC. AD = 3 cm, DB = 9 cm and the area of triangle ADE is 6 cm^2.
Work out the area of the trapezium DBCE.
The diagram is not drawn accurately.

Triangles ADE and ABC are similar. AB = 3 + 9 = 12 cm
Length scale factor = 12 ÷ 3 = 4
Area scale factor = $(4)^2 = 16$
Area of triangle ABC = 6 × 16 = 96 cm^2
Trapezium = 96 − 6 = 90 cm^2
Answer: 90 cm^2

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6 Equation of a circle

A and B are the points $(-1, 1)$ and $(-7, -5)$.
AB is a diameter of a circle.
Find an equation of the circle.

The centre is the midpoint of AB: $(\frac{-1+(-7)}{2}, \frac{1+(-5)}{2}) = (-4, -2)$

$r^2 = (-1-(-4))^2 + (1-(-2))^2 = 9 + 9 = 18$

$(x+4)^2 + (y+2)^2 = 18$

Answer: $(x+4)^2 + (y+2)^2 = 18$

7 HCF and LCM

The HCF of two numbers is 3 and their LCM is 180.
The smaller number is 15.
Work out the larger number.

HCF × LCM = product of the two numbers

$3 \times 180 = 540$

Larger number $= 540 \div 15 = 36$

Answer: 36

8 Area and quadratic equations

The diagram shows a rectangle of length $(3x + 5)$ cm, width $(2x - 1)$ cm and area 253 cm^2.
(a) Show that $6x^2 + 7x - 258 = 0$.
(b) Work out the perimeter of the rectangle.

(a) $(3x + 5)(2x - 1) = 253$

Expand: $6x^2 + 7x - 5 = 253$, so $6x^2 + 7x - 258 = 0$

(b) Factorise: $(x - 6)(6x + 43) = 0$

$x = 6$ ($x = -\frac{43}{6}$ is not possible as a length would be negative)

Length = 23 cm and width = 11 cm

Perimeter $= 2 \times (23 + 11) = 68$ cm

Answer: (b) 68 cm

9 Finding the rate of interest

Leila invested £10,500 in a savings account for 2 years.
Compound interest was paid at a fixed rate per year.
At the end of the 2 years the account held £12,247.20.
Work out the annual rate of interest. (Calculator)

Let the yearly multiplier be m, so $10500 \times m^2 = 12247.20$

$m^2 = 12247.20 \div 10500 = 1.1664$

$m = \sqrt{1.1664} = 1.08$

Rate $= (1.08 - 1) \times 100 = 8\%$

Answer: 8% per year

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10 Percentages with an unknown rate

The price of a ticket is decreased by x%.
The new price is then increased by x%.
The final price is 25% less than the original price.
Work out the value of x.

Let the original price be 1. Multipliers: $(1 + \frac{x}{100})$ and $(1 - \frac{x}{100})$

$(1 + \frac{x}{100})(1 - \frac{x}{100}) = 1 - \frac{x^2}{10000}$

Final price is 25% less, so the multiplier is 0.75: $1 - \frac{x^2}{10000} = 0.75$

$\frac{x^2}{10000} = 0.25$, so $x^2 = 2500$ and $x = 50$

Answer: x = 50