Day 336 **S Tier 10-a-day: Model Solutions**

1 Comparing costs

Hire Shop A has a delivery charge of £5 plus £5.50 per day.
Hire Shop B has a delivery charge of £23 plus £2.50 per day.
(a) Work out the number of days for which the two costs are the same.
(b) Which is cheaper for 11 days, and by how much?

(a) Let n be the number of days. Working in pounds, the costs are equal when $5 + 5.50n = 23 + 2.50n$
$3n = 18$, so $n = 6$
(b) Hire Shop A: £5 + £5.50 × 11 = £65.50
Hire Shop B: £23 + £2.50 × 11 = £50.50
Hire Shop B is cheaper by £15
Answer: (a) 6 (b) Hire Shop B, by £15

2 Finding a percentage rate

A machine was bought for £1,600.
Each year its value decreases by the same percentage.
After 2 years it is worth £400.
Work out the percentage decrease each year.

Multiplier for 2 years = £400 ÷ £1,600 = 0.25
Yearly multiplier = the square root of 0.25 = 0.5
50% decrease each year
Answer: 50%

3 Factorising fully

Factorise fully $(x + 4)^2 - 81$

This is a difference of two squares: $(x + 4)^2 - 9^2$
$= (x + 4 - 9)(x + 4 + 9)$
$= (x - 5)(x + 13)$
Answer: $(x - 5)(x + 13)$

4 Standard form

Work out $(5.4 \times 10^{-2}) + (8.1 \times 10^{-5})$
Give your answer in standard form.

Write the first number as a multiple of 10^{-5}: $5.4 \times 10^{-2} = 5400 \times 10^{-5}$
$(5400 + 8.1) \times 10^{-5} = 5408.1 \times 10^{-5}$
$= 5.4081 \times 10^{-2}$
Answer: 5.4081×10^{-2}

5 Equation of a circle

A and B are the points (−2, 8) and (−10, 12).
AB is a diameter of a circle.
Find an equation of the circle.

The centre is the midpoint of AB: $(\frac{-2 + (-10)}{2}, \frac{8 + 12}{2}) = (-6, 10)$
$r^2 = (-2 - (-6))^2 + (8 - 10)^2 = 16 + 4 = 20$
$(x + 6)^2 + (y - 10)^2 = 20$
Answer: $(x + 6)^2 + (y - 10)^2 = 20$

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6 Fractions removed from a ratio

The ratio of blue to green sweets in a tin is 9 : 4.

Then $\frac{1}{3}$ of the blue and $\frac{1}{5}$ of the green are taken out.

Work out the ratio of blue to green now.

Left: $\frac{2}{3} \times 9 = 6$ of the blue part, $\frac{4}{5} \times 4 = \frac{16}{5}$ of the green part

$6 : \frac{16}{5}$

Multiply by 5: 30 : 16 = 15 : 8

Answer: 15 : 8

7 Bounds of roots

The area of a circle is 93 cm^2, correct to the nearest cm^2.

Work out the lower bound of the radius of the circle.

Give your answer correct to 3 significant figures. (Calculator)

$92.5 \leq$ area < 93.5

Area = πr^2, so r = $\sqrt{\frac{\text{area}}{\pi}}$

Lower bound = $\sqrt{\frac{92.5}{\pi}}$ = 5.42620... = 5.43 (3 s.f.)

Answer: 5.43 cm

8 Pythagoras with algebra

ABC is a right-angled triangle with sides (x − 3) cm, (x + 4) cm and (2x − 3) cm. The longest side is (2x − 3) cm.

(a) Find the value of x.

(b) Work out the area of the triangle.

The diagram is not drawn accurately.

(a) Pythagoras: $(x - 3)^2 + (x + 4)^2 = (2x - 3)^2$

Expand and collect terms: $2x^2 - 14x - 16 = 0$

Factorise: $2(x - 8)(x + 1) = 0$

x = 8 or x = −1, and x must be positive, so x = 8

(b) The two shorter sides are 5 cm and 12 cm

Area = $\frac{1}{2} \times 5 \times 12 = 30$ cm^2

Answer: (a) x = 8 (b) 30 cm^2

9 Venn diagrams with algebra

The Venn diagram shows how many of 49 students have a tablet (T) or a laptop (L).

(a) Work out the value of x.

(b) A student is chosen at random from those who have a laptop. Work out the probability that this student also has a tablet.

(a) Total: (x + 5) + x + (2x) + 8 = 49

4x + 13 = 49, so x = 9

(b) Students in the second circle: x + 2x = 9 + 18 = 27, of whom 9 are in both

Probability = $\frac{1}{3}$

Answer: (a) x = 9 (b) $\frac{1}{3}$

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10 Loci and regions

ABCD is a rectangle with AB = 16 cm and AD = 14 cm.
A point P inside the rectangle is such that AP ≤ 10 cm and P is nearer to AB than to AD.
Work out the area of the region in which P can lie. Give your answer in terms of π.

AP ≤ 10 cm: P is inside the quarter circle of radius 10 cm centred at A (it fits inside the rectangle)
Nearer to AB than AD: P is on the AB side of the angle bisector of angle DAB, which is the line through A at 45° to AB
The bisector cuts the quarter circle into two equal halves

Area = $\frac{1}{2} \times \frac{1}{4} \times \pi \times 10^2 = \frac{25}{2}\pi$ cm^2

Answer: $\frac{25}{2}\pi$ cm^2