Day 335 **S Tier 10-a-day: Model Solutions**

1 Equations with fractions

Solve $\frac{x+4}{5} - \frac{x-6}{3} = 2$

Multiply every term by 15: $3(x + 4) - 5(x - 6) = 30$
Expand: $-2x + 42 = 30$
$-2x = -12$
$x = 6$
Answer: x = 6

2 Alternate segment theorem

PAQ is a tangent to the circle at A. B and C are points on the circle. Angle PAB = 74° and angle CAQ = 52°.
(a) Work out the size of angle ABC.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) Angle ABC = angle CAQ = 52° (alternate segment theorem)
(b) Angle ACB = angle PAB = 74° (alternate segment theorem)
Answer: (a) 52° (b) 74°

3 Expanding three brackets

Expand and simplify $(x + 1)(x + 6)(x - 3)$

Expand the first two brackets: $(x + 1)(x + 6) = x^2 + 7x + 6$
Multiply by $(x - 3)$: $x(x^2 + 7x + 6) - 3(x^2 + 7x + 6)$
$= x^3 + 4x^2 - 15x - 18$
Answer: $x^3 + 4x^2 - 15x - 18$

4 Direct proportion formulae

A is directly proportional to r.
When r = 6, A = 48.
(a) Find a formula for A in terms of r.
(b) Work out the value of A when r = 1.

(a) A = kr
48 = k × 6, so k = 48 ÷ 6 = 8
A = 8r
(b) A = 8 × 1 = 8
Answer: (a) A = 8r (b) 8

5 Fractional and negative indices

Work out $\left(\frac{121}{4}\right)^{-\frac{3}{2}}$

A negative power means take the reciprocal: $\left(\frac{121}{4}\right)^{-\frac{3}{2}} = \left(\frac{4}{121}\right)^{\frac{3}{2}}$

The denominator 2 means the square root: the square root of $\frac{4}{121}$ is $\frac{2}{11}$

The numerator 3 means raise to the power 3: $\left(\frac{2}{11}\right)^3 = \frac{8}{1331}$

Answer: $\frac{8}{1331}$

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6 Sine rule

The diagram shows triangle ABC. (Calculator)
Work out the length of AC. Give your answer correct to 1 decimal place.
The diagram is not drawn accurately.

Use the sine rule: $\frac{AC}{\sin B} = \frac{BC}{\sin A}$

$\frac{x}{\sin 84°} = \frac{8}{\sin 63°}$

$x = \frac{8 \times \sin 84°}{\sin 63°} = 8.9294$

x = 8.9 cm (1 d.p.)

Answer: AC = 8.9 cm

7 Reverse percentages

The value of a house is increased by 40% in one year.
The next year its value is decreased by 40%.
It is now worth £252,000.
Work out the value of the house before the first change.

Multipliers: 1.4, 0.6
Overall multiplier = 1.4 × 0.6 = 0.84
Original = final ÷ multiplier = £252,000 ÷ 0.84
= £300,000

Answer: £300,000

8 Density of an alloy

An alloy contains 284 g of zinc and 89 g of copper.
The density of zinc is 7.1 g/cm^3. The density of copper is 8.9 g/cm^3.
Work out the density of the alloy.

Volume of zinc = 284 ÷ 7.1 = 40 cm^3
Volume of copper = 89 ÷ 8.9 = 10 cm^3
Total mass = 373 g, total volume = 50 cm^3
Density = 373 ÷ 50 = 7.46 g/cm^3

Answer: 7.46 g/cm^3

9 Loci and regions

ABCD is a rectangle with AB = 8 cm and AD = 9 cm.
A point P inside the rectangle is such that AP ≤ 8 cm and P is nearer to A than to B.
Work out the area of the region in which P can lie. Give your answer in terms of π and surds.

Nearer to A than to B: P is on the A side of the perpendicular bisector of AB, a distance 4 cm from A
AP ≤ 8 cm: P is inside the quarter circle of radius 8 cm centred at A

The bisector meets the arc where $\cos\theta = \frac{4}{8}$, so θ = 60° at A. The region is a right-angled triangle plus a sector of angle 30°

Triangle: base 4 cm, height 8 sin 60° = $4\sqrt{3}$ cm, area = $\frac{1}{2} \times 4 \times 4\sqrt{3} = 8\sqrt{3}$

Sector of angle 30° : $\frac{30}{360} \times \pi \times 8^2 = \frac{16}{3}\pi$

Area = $8\sqrt{3} + \frac{16}{3}\pi$ cm^2

Answer: $(8\sqrt{3} + \frac{16}{3}\pi)$ cm^2

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10 Probability from a frequency tree

In a population of 2,000 people, 5% have a certain disease.
A test gives a positive result for 80% of people who have the disease and for 10% of people who do not.
A person is chosen at random and tests positive.
Work out the probability that they have the disease.

5% of 2,000 = 100 have the disease and 1,900 do not
Flagged and genuine: 100 × 80% = 80
Flagged but not: 1,900 × 10% = 190
Total flagged or positive = 80 + 190 = 270

Probability = 80 ÷ 270 = $\frac{8}{27}$

Answer: $\mathbf{\frac{8}{27}}$