Day 334 **S Tier 10-a-day: Model Solutions**

1 Rationalising the denominator

Rationalise the denominator of $\frac{20}{3\sqrt{2}}$

Give your answer in its simplest form.

Multiply top and bottom by $\sqrt{2}$:

$\frac{20}{3\sqrt{2}} = \frac{20\sqrt{2}}{6}$

$= \frac{10\sqrt{2}}{3}$

Answer: $\mathbf{\frac{10\sqrt{2}}{3}}$

2 Enlargement

Triangle P has vertices A(4, 1), B(2, 4) and C(6, 3).
Triangle Q has vertices A'(−8, −11), B'(−2, −20) and C'(−14, −17).
Q is an enlargement of P.
Describe fully the single transformation that maps P onto Q.

x-step from A to B = −2, x-step from A' to B' = 6
Scale factor = 6 ÷ −2 = −3
The centre C satisfies: C + −3(A − C) = A', so (1 − −3)C = A' − −3 A
x: 4 × x = (−8) − (−3) × 4 = 4, so x = 1
y: 4 × y = (−11) − (−3) × 1 = −8, so y = −2
Answer: Enlargement, scale factor −3, centre (1, −2)

3 Perpendicular bisector

A is the point (8, 5) and B is the point (18, 1).
Find an equation of the perpendicular bisector of AB.
Give your answer in the form y = mx + c.

Midpoint of AB = $(\frac{8+18}{2}, \frac{5+1}{2}) = (13, 3)$

Gradient of AB = $\frac{1-5}{18-8} = -\frac{2}{5}$

The perpendicular gradient is $-1 \div (-\frac{2}{5}) = \frac{5}{2}$

Using the midpoint: $y - 3 = \frac{5}{2}(x - 13)$

$y = \frac{5}{2}x - \frac{59}{2}$

Answer: $\mathbf{y = \frac{5}{2}x - \frac{59}{2}}$

4 Exchange rates

Carlos changes £400 into dollars at £1 = $1.2.
Carlos spends $270 and then changes the rest back into pounds at £1 = $1.4.
How many pounds does Carlos get back?

£400 × 1.2 = $480
480 − 270 = $210
210 ÷ 1.4 = £150
Answer: £150

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5 Fraction problems

A bag contains sweets.

$\frac{1}{4}$ of the sweets are orange and $\frac{1}{3}$ of the sweets are lemon.

The rest of the sweets, 40 of them, are strawberry.

How many lemon sweets are there?

Fraction that are strawberry = $1 - \frac{1}{4} - \frac{1}{3} = \frac{5}{12}$

$\frac{5}{12}$ of the total is 40

Total = $40 \div \frac{5}{12} = 96$

Lemon sweets = $\frac{1}{3}$ of 96 = 32

Answer: 32

6 Sample space and conditional probability

A spinner numbered 1 to 4 and a spinner numbered 1 to 5 are each thrown once. The first score and the second score are recorded.

(a) Work out the probability that the product is even.

(b) Given that at least one number is 3, work out the probability that the product is even.

(a) There are $4 \times 5 = 20$ equally likely outcomes, and 14 of them satisfy: the product is even

Probability = $\frac{7}{10}$

(b) 8 outcomes satisfy: at least one number is 3. Of these, 4 also satisfy: the product is even

Probability = $\frac{1}{2}$

Answer: (a) $\frac{7}{10}$ (b) $\frac{1}{2}$

7 Pythagoras with algebra

ABC is a right-angled triangle with sides (2x + 6) cm, (x + 1) cm and (2x + 8) cm. The longest side is (2x + 8) cm.

(a) Find the value of x.

(b) Work out the area of the triangle.

The diagram is not drawn accurately.

(a) Pythagoras: $(2x + 6)^2 + (x + 1)^2 = (2x + 8)^2$

Expand and collect terms: $x^2 - 6x - 27 = 0$

Factorise: $(x - 9)(x + 3) = 0$

$x = 9$ or $x = -3$, and x must be positive, so $x = 9$

(b) The two shorter sides are 24 cm and 10 cm

Area = $\frac{1}{2} \times 24 \times 10 = 120$ cm^2

Answer: (a) x = 9 (b) 120 cm^2

8 Algebraic fractions: multiplying and dividing

Simplify fully $\frac{x^2 - 64}{x^2 - 2x} \div \frac{x^2 - 13x + 40}{x}$

Factorise: $\frac{(x - 8)(x + 8)}{x(x - 2)} \div \frac{(x - 5)(x - 8)}{x}$

Turn the second fraction upside down and multiply: $\frac{(x - 8)(x + 8)}{x(x - 2)} \times \frac{x}{(x - 5)(x - 8)}$

Cancel $(x - 8)$ and x

$= \frac{(x + 8)}{(x - 5)(x - 2)}$

Answer: $\frac{(x + 8)}{(x - 5)(x - 2)}$

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9 Transformations of graphs

The graph of $y = f(x)$ has a maximum point at $(-1, 1)$.
Write down the coordinates of the turning point of the graph of
(a) $y = f(x - 2)$
(b) $y = 3f(x + 2)$

(a) Start with the turning point $(-1, 1)$
Apply the transformation to the point: $(1, 1)$
(b) $f(x + 2)$ changes the x-coordinate to -3, and $3f(x)$ changes the y-coordinate to 3
Turning point: $(-3, 3)$
Answer: (a) (1, 1) (b) (−3, 3)

10 Tangent to a circle: proof

Show that the line $y = x - 12$ is a tangent to the circle $x^2 + y^2 = 72$.
Find the coordinates of the point where the line touches the circle.

Substitute $y = x - 12$ into $x^2 + y^2 = 72$: $x^2 + (x - 12)^2 = 72$
Expand: $2x^2 - 24x + 72 = 0$
$b^2 - 4ac = (-24)^2 - 4 \times 2 \times 72 = 576 - 576 = 0$
$b^2 - 4ac = 0$, so there is exactly one point of intersection: the line is a tangent
$x = \frac{-(-24)}{2 \times 2} = 6$, and $y = 6 - 12 = -6$. The point is $(6, -6)$
Answer: (6, −6)