Day 333 **S Tier 10-a-day: Model Solutions**

1 Pythagoras and quadratic equations

The lengths of the sides of a right-angled triangle are $(x - 2)$ cm, x cm and $(x + 2)$ cm.
The longest side is $(x + 2)$ cm.
(a) Show that $x^2 - 8x = 0$
(b) Hence work out the area of the triangle.

(a) Pythagoras: $(x - 2)^2 + x^2 = (x + 2)^2$
Expand: $x^2 - 4x + 4 + x^2 = x^2 + 4x + 4$
Rearrange: $x^2 - 8x = 0$
(b) Factorise: $(x - 8)x = 0$, so $x = 8$ (the other root, $x = 0$, does not give valid side lengths)
The shorter sides are 6 cm and 8 cm, so area $= \frac{1}{2} \times 6 \times 8 = 24$ cm^2

Answer: (b) 24 cm^2

2 Arc length and sector area

A sector of a circle has radius 8 cm and angle 72°.
(a) Work out the length of the arc of the sector.
(b) Work out the area of the sector. Give each answer in terms of π.

(a) Fraction of the circle $= \frac{72}{360}$
Arc length $= \frac{72}{360} \times 2 \times \pi \times 8 = \frac{16}{5}\pi$ cm
(b) Area $= \frac{72}{360} \times \pi \times 8^2 = \frac{64}{5}\pi$ cm^2

Answer: (a) $\frac{16}{5}\pi$ cm (b) $\frac{64}{5}\pi$ cm^2

3 Expanding brackets with surds

Expand and simplify $(1 + \sqrt{3})(4 + \sqrt{3})$
Give your answer in the form $p + q\sqrt{3}$

$(1 + \sqrt{3})(4 + \sqrt{3}) = 4 + \sqrt{3} + 4\sqrt{3} + 3$
$= 7 + 5\sqrt{3}$

Answer: $7 + 5\sqrt{3}$

4 Similar shapes

Two shapes are mathematically similar.
The smaller has height 18 cm and area 36 cm^2.
The larger has area 81 cm^2.
Work out the height of the larger shape.

Ratio of areas = 81 : 36 = 9 : 4
Ratio of lengths = 3 : 2
Height $= 18 \times \frac{3}{2} = 27$ cm

Answer: 27 cm

5 Estimating the mean from a histogram table

The table gives information about the times of some people.
Work out an estimate for the mean time.
Give your answer correct to 1 decimal place. (Calculator)

Frequency = frequency density × class width: 20, 4, 6, 8
Mid-points: 15, 25, 40, 60
$\Sigma fm = 20 \times 15 + 4 \times 25 + 6 \times 40 + 8 \times 60 = 1120$, $\Sigma f = 38$
Mean $= 1120 \div 38 = 29.5$
Answer: 29.5

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6 Composite functions: fg and gf

$f(x) = 4x + 1$ and $g(x) = x^2$
Solve $fg(x) = gf(x)$

$fg(x) = f(x^2) = 4x^2 + 1$
$gf(x) = g(4x + 1) = (4x + 1)^2 = 16x^2 + 8x + 1$
Set them equal and rearrange: $12x^2 + 8x = 0$
Divide by 4 and factorise: $x(3x + 2) = 0$
$x = -\frac{2}{3}$ or $x = 0$

Answer: $x = -\frac{2}{3}$ or $x = 0$

7 Area and quadratic equations

The diagram shows a rectangle of length $(x + 1)$ cm, width $(x - 2)$ cm and area 54 cm^2.
(a) Show that $x^2 - x - 56 = 0$.
(b) Work out the perimeter of the rectangle.

(a) $(x + 1)(x - 2) = 54$
Expand: $x^2 - x - 2 = 54$, so $x^2 - x - 56 = 0$
(b) Factorise: $(x - 8)(x + 7) = 0$
$x = 8$ ($x = -7$ is not possible as a length would be negative)
Length = 9 cm and width = 6 cm
Perimeter = $2 \times (9 + 6) = 30$ cm
Answer: (b) 30 cm

8 Ratio and algebra

In a fruit box, the ratio of the number of apples to the number of pears is 2 : 7.
Then 2 more apples are added and 6 pears are removed.
The ratio is now 1 : 3.
How many pears were there at the start?

Let the numbers at the start be $2k$ and $7k$
$(2k + 2) : (7k - 6) = 1 : 3$, so $3(2k + 2) = 1(7k - 6)$
$6k + 6 = 7k - 6$
$-1k = -12$, so $k = 12$
Number of pears at the start: $7 \times 12 = 84$
Answer: 84

9 Squares, cubes and prime factors

Find the smallest positive integer k such that $168 \times k$ is a cube number.

$168 = 2^3 \times 3 \times 7$
For a cube number every power must be a multiple of 3
Multiply by $3^2 \times 7^2$ to get $2^3 \times 3^3 \times 7^3$
$k = 441$
Answer: $k = 441$

10 Joint and inverse variation

y is directly proportional to the square of x and inversely proportional to z.
When x = 6 and z = 2, y = 108.
Work out the value of y when x = 6 and z = 4.

$y = \frac{kx^2}{z}$

$108 = \frac{k \times 36}{2}$, so $k = 108 \times 2 \div 36 = 6$

$y = \frac{6 \times 36}{4} = 54$

Answer: $y = 54$