Day 332 **S Tier 10-a-day: Model Solutions**

1 Angles with algebra

In triangle PQR, side QR is extended to S.
Angle QPR = (2x + 25)°
Angle PQR = (2x − 15)°
Angle PRS = (2x + 48)°
(a) Work out the value of x.
(b) Work out the size of angle PRQ.

(a) The exterior angle of a triangle equals the sum of the two opposite interior angles
2x + 48 = 4x + 10
2x = 38
x = 19
(b) Angle PRS = 86° so angle PRQ = 180° − 86° = 94°
Answer: (a) x = 19 (b) 94°

2 Trigonometric graphs

The diagram shows the graph of y = a sin x° + c for 0 ≤ x ≤ 360.
Some points on the graph are labelled.
Find the value of a and the value of c.

At x = 90, sin x° = 1, so a + c = 2
At x = 270, sin x° = −1, so −a + c = 6
Add: 2c = 8, so c = 4
Subtract: 2a = −4, so a = −2
Answer: a = −2, c = 4

3 Rationalising the denominator

Rationalise the denominator of $\frac{21}{3\sqrt{6}}$
Give your answer in its simplest form.

Multiply top and bottom by $\sqrt{6}$:
$\frac{21}{3\sqrt{6}} = \frac{21\sqrt{6}}{18}$
$= \frac{7\sqrt{6}}{6}$

Answer: $\frac{7\sqrt{6}}{6}$

4 Quadratic graphs

The curve $y = 2x^2 - 32x + 96$ crosses the x-axis at the points A and B.
(a) Find the coordinates of A and B.
(b) Find the coordinates of the minimum point of the curve.

(a) Put y = 0. $2x^2 - 32x + 96 = 0$, so $2(x^2 + 16x + 48) = 0$, giving $2(x - 4)(x - 12) = 0$
x − 4 = 0 or x − 12 = 0, so x = 4 or x = 12
(b) The turning point is halfway between A and B: x = (4 + 12) ÷ 2 = 8
When x = 8: $y = 2 \times 8^2 - 32 \times 8 + 96 = -32$
Answer: (a) A(4, 0), B(12, 0) (b) (8, −32)

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5 Pressure

A stone block in the shape of a cuboid weighs 690 N.
Its smallest face has area 1000 cm^2. Its largest face has area 2,000 cm^2.
Pressure = force ÷ area
Work out (a) the greatest pressure and (b) the least pressure the stone block can exert on the ground, in N/m^2.

1 m^2 = 10 000 cm^2, so 1000 cm^2 = 0.1 m^2 and 2,000 cm^2 = 0.2 m^2
(a) Greatest pressure is on the smallest face: 690 ÷ 0.1 = 6,900 N/m^2
(b) Least pressure is on the largest face: 690 ÷ 0.2 = 3,450 N/m^2
Answer: (a) 6,900 N/m^2 (b) 3,450 N/m^2

6 Recurring decimals

Convert the recurring decimal $0.\dot{6}$ to a fraction.
Give your fraction in its simplest form.

Let x = 0.6666...
10x = 6.6666...
Subtract: 9x = 6
$x = \frac{6}{9} = \frac{2}{3}$
Answer: $\frac{2}{3}$

7 Vector proof: collinear points

OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
OP = 3$\mathbf{a}$, OQ = 15$\mathbf{a}$ + 9$\mathbf{b}$ and OR = 31$\mathbf{a}$ + 21$\mathbf{b}$.
(a) Show that P, Q and R lie on a straight line.
(b) Work out the ratio PQ : QR in its simplest form.

(a) PQ = OQ − OP = 12$\mathbf{a}$ + 9$\mathbf{b}$
QR = OR − OQ = 16$\mathbf{a}$ + 12$\mathbf{b}$
$QR = \frac{4}{3}$ PQ, so PQ and QR are parallel, and they share the point Q
So P, Q and R lie on a straight line
(b) PQ : QR = 3 : 4
Answer: (b) 3 : 4

8 Speed-time graphs

The graph shows the speed of a train between two stations.
(a) Work out the acceleration in the first 10 seconds.
(b) Work out the total distance.

(a) Acceleration = change in speed ÷ time = 20 ÷ 10 = 2 m/s^2
(b) Distance = area under the graph, split into two triangles and a rectangle
$\frac{1}{2} \times 10 \times 20 + 20 \times 20 + \frac{1}{2} \times 20 \times 20 = 100 + 400 + 200$
= 700
Answer: (a) 2 m/s^2 (b) 700 m

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9 Angles of elevation

A point A is on level ground in line with the base of a cliff. (Calculator)
The angle of elevation of the top of the cliff from A is 37°.
B is a point on the same line, 50 m further from the cliff than A.
The angle of elevation of the top of the cliff from B is 19°.
Work out the height of the cliff. Give your answer to 1 decimal place.

Let the height be h and the distance from A to the base be x. $\tan 37° = \frac{h}{x}$, so $x = h \div \tan 37°$

From B: $\tan 19° = \frac{h}{x + 50}$, so $x + 50 = h \div \tan 19°$

Subtract: $50 = h \div \tan 19° - h \div \tan 37° = h(2.9042 - 1.3270)$
$h = 50 \div 1.5772 = 31.7$ m (1 d.p.)
Answer: 31.7 m

10 Exponential growth and decay

There are 25 bacteria in a sample. The number of bacteria is multiplied by 4 every day.
After how many days will there first be more than 12,500 bacteria?

Number after n days = 25×4^n
n = 4: 6,400
n = 5: 25,600
25,600 is the first value above 12,500
Answer: 5 days