Day 331 **S Tier 10-a-day: Model Solutions**

1 Factorising quadratics

Factorise $9x^2 - 9x + 2$

$9 \times 2 = 18$. Two numbers that multiply to 18 and add to -9 are -3 and -6

Split the middle term: $9x^2 - 3x - 6x + 2$

Factorise in pairs: $3x(3x - 1) - 2(3x - 1)$

$= (3x - 2)(3x - 1)$

Answer: $(3x - 2)(3x - 1)$

2 Profit and loss

A shop buys a bike for £140.
It adds a 30% mark-up to get the selling price.
In a sale the selling price is reduced by 15%.
Work out the percentage profit or loss on the original cost.

Marked price = £140 × 1.3 = £182

Sale price = £182 × 0.85 = £154.70

Profit = £14.70

Percentage = $\frac{14.7}{140} \times 100 = 10.5\%$

Answer: 10.5% profit

3 Equations with fractions

Solve $\frac{x - 3}{2} + \frac{x - 6}{3} = -6$

Multiply every term by 6: $3(x - 3) + 2(x - 6) = -36$

Expand: $5x - 21 = -36$

$5x = -15$

$x = -3$

Answer: $x = -3$

4 Combined transformations

The point P(−1, 2) is reflected in the line $y = x$.
The image is then rotated 180° about the origin.
(a) Write down the coordinates of the final image of P.
(b) Describe fully the single transformation that is equivalent to these two transformations.

(a) After the first transformation: P' = (2, −1)

After the second transformation: P'' = (−2, 1)

(b) Follow a general point (x, y) through both transformations: $(x, y) \to (y, x) \to (-y, -x)$

The single mapping is $(x, y) \to (-y, -x)$

This is a reflection in the line $y = -x$

Answer: (a) (−2, 1) (b) Reflection in the line $y = -x$

5 Expanding brackets with surds

Show that $(2 + 3\sqrt{3})(2 - 3\sqrt{3})$ is an integer.

Use the difference of two squares: $(2 + 3\sqrt{3})(2 - 3\sqrt{3}) = 4 - (3\sqrt{3})^2$

$= 4 - 27 = -23$

Answer: -23

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6 Fractions removed from a ratio

The ratio of men to women people at a meeting is 9 : 8.

Then $\frac{2}{3}$ of the men and $\frac{1}{4}$ of the women are taken out.

Work out the ratio of men to women now.

Left: $\frac{1}{3} \times 9 = 3$ of the men part, $\frac{3}{4} \times 8 = 6$ of the women part

3 : 6

Multiply by 1: 3 : 6 = 1 : 2

Answer: 1 : 2

7 Recurring decimals

Without using a calculator, work out $0.\dot{8}\dot{2} \times 0.\dot{4}\dot{0}$

Give your answer as a fraction in its simplest form.

$0.\dot{8}\dot{2} = \frac{82}{99}$

$0.\dot{4}\dot{0} = \frac{40}{99}$

$\frac{82}{99} \times \frac{40}{99} = \frac{3280}{9801}$

Answer: $\frac{3280}{9801}$

8 Angles of elevation

A point A is on level ground in line with the base of a tower. (Calculator)

The angle of elevation of the top of the tower from A is 46°.

B is a point on the same line, 25 m further from the tower than A.

The angle of elevation of the top of the tower from B is 19°.

Work out the height of the tower. Give your answer to 1 decimal place.

Let the height be h and the distance from A to the base be x. $\tan 46° = \frac{h}{x}$, so $x = h \div \tan 46°$

From B: $\tan 19° = \frac{h}{x + 25}$, so $x + 25 = h \div \tan 19°$

Subtract: $25 = h \div \tan 19° - h \div \tan 46° = h(2.9042 - 0.96569)$

$h = 25 \div 1.9385 = 12.9$ m (1 d.p.)

Answer: 12.9 m

9 Independent events and probability rules

A and B are two events.

P(A) = 0.2, P(B) = 0.5 and P(A or B) = 0.6

(a) Work out P(A and B).

(b) Are A and B independent? You must show your working.

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.6 = 0.2 + 0.5 - P(A \cap B)$, so $P(A \cap B) = 0.1$

(b) $P(A) \times P(B) = 0.2 \times 0.5 = 0.1$

This equals $P(A \cap B)$, so A and B are independent

Answer: (a) 0.1 (b) Yes

10 Solving trigonometric equations

Solve $\sin x = \frac{1}{2}$ for $0° \leq x \leq 360°$.

Do not use a calculator.

The related acute angle satisfies $\sin \theta = \frac{1}{2}$, so $\theta = 30°$

sine is positive in the 1st and 2nd quadrants, so $x = 30°$ and $x = 180° - 30°$

$x = 30°$ and $x = 150°$

Answer: x = 30° or x = 150°