Day 330 **S Tier 10-a-day: Model Solutions**

1 Error intervals

The width of a table is 24,100 cm, correct to 3 significant figures.
Write down the error interval for the width of a table, using w.

The value has been rounded to the nearest 100
Half of this is 50, so go 50 below and 50 above
24,100 − 50 = 24050 and 24,100 + 50 = 24150
24050 ≤ w < 24150
Answer: 24050 ≤ w < 24150

2 Inverse proportion in context

20 painters can paint a hall in 8 days.
After 5 days, 5 of the painters leave. The rest carry on working at the same rate.
How many more days does it take to finish?

Total work = 20 × 8 = 160 painter-days
Work done = 20 × 5 = 100, so work left = 60
Workers left = 20 − 5 = 15
Time = 60 ÷ 15 = 4 days
Answer: 4 days

3 Standard form

Work out $(3.8 \times 10^{-5}) \times (1.3 \times 10^{-6})$
Give your answer in standard form.

3.8 × 1.3 = 4.94
$10^{-5} \times 10^{-6} = 10^{-11}$
$= 4.94 \times 10^{-11}$
Answer: 4.94×10^{-11}

4 Quadratic sequences

Here are the first five terms of a quadratic sequence.
0, 1, −2, −9, −20
Find an expression for the nth term.

First differences: 1, −3, −7, −11
Second differences: −4, −4, −4, so a = −4 ÷ 2 = −2
Subtract $-2n^2$ from each term: 2, 9, 16, 23, 30
The remaining linear sequence has nth term 7n − 5
nth term = $-2n^2 + 7n - 5$
Answer: $-2n^2 + 7n - 5$

5 Areas of similar triangles

In the diagram, DE is parallel to BC. AD = 6 cm, DB = 8 cm and the area of triangle ADE is 180 cm^2.
Work out the area of the trapezium DBCE.
The diagram is not drawn accurately.

Triangles ADE and ABC are similar. AB = 6 + 8 = 14 cm
Length scale factor = $14 \div 6 = \frac{7}{3}$
Area scale factor = $(\frac{7}{3})^2 = \frac{49}{9}$
Area of triangle ABC = $180 \times \frac{49}{9} = 980$ cm^2
Trapezium = 980 − 180 = 800 cm^2
Answer: 800 cm^2

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6 Simple and compound interest

Ravi invests £10,000 for 3 years at 4.5% per year simple interest.
Sofia invests £10,000 for 3 years at 4.5% per year compound interest.
Work out how much more interest Sofia earns than Ravi.
Give your answer to the nearest penny. (Calculator)

Simple interest: £10,000 × 4.5% × 3 = £1,350

Compound: £10,000 × 1.045^3 = £11,411.66

Compound interest = £11,411.66 − £10,000 = £1,411.66

Difference = £1,411.66 − £1,350 = £61.66

Answer: £61.66

7 Perpendicular bisector

A is the point (4, −5) and B is the point (12, −7).
Find an equation of the perpendicular bisector of AB. Give your answer in the form ax + by = c, where a, b and c are integers.

Midpoint of AB = (8, −6)

Gradient of AB = $-\frac{1}{4}$

Perpendicular gradient = 4

y − (−6) = 4(x − 8)

Rearrange to integer coefficients: 4x − y = 38

Answer: 4x − y = 38

8 Algebraic fractions: multiplying and dividing

Simplify fully $\frac{x^2 - 6x - 7}{x^2 + 3x} \times \frac{x^2 + 4x}{x - 7}$

Factorise: $\frac{(x - 7)(x + 1)}{x(x + 3)} \times \frac{x(x + 4)}{(x - 7)}$

Cancel (x − 7) and x

= $\frac{(x + 1)(x + 4)}{(x + 3)}$

Answer: $\mathbf{\frac{(x + 1)(x + 4)}{(x + 3)}}$

9 Sample space and conditional probability

A spinner numbered 1 to 4 and a spinner numbered 1 to 5 are each thrown once. The first score and the second score are recorded.
(a) Work out the probability that the product is greater than 8.
(b) Given that the first number is greater than the second, work out the probability that the product is greater than 8.

(a) There are 4 × 5 = 20 equally likely outcomes, and 7 of them satisfy: the product is greater than 8

Probability = $\frac{7}{20}$

(b) 6 outcomes satisfy: the first number is greater than the second. Of these, 1 also satisfy: the product is greater than 8

Probability = $\frac{1}{6}$

Answer: (a) $\mathbf{\frac{7}{20}}$ **(b)** $\mathbf{\frac{1}{6}}$

10 Equal roots and real roots

Show that the equation $x^2 + kx - 3 = 0$ has two distinct real solutions for all values of k.

$b^2 - 4ac = k^2 - 4 \times 1 \times (-3)$

$= k^2 + 12$

k^2 is never negative, so $k^2 + 12$ is always at least 12, which is greater than 0

So $b^2 - 4ac > 0$ for all k, and there are always two distinct real solutions

Answer: Shown