Day 33 **S Tier 10-a-day: Model Solutions**

1 Finding the rate of interest

Jamal invested £18,500 in a savings account for 3 years.
Compound interest was paid at a fixed rate per year.
At the end of the 3 years the account held £24,623.50.
Work out the annual rate of interest. (Calculator)

Let the yearly multiplier be m, so $18500 \times m^3 = 24623.50$
$m^3 = 24623.50 \div 18500 = 1.331$
$m = \sqrt[3]{1.331} = 1.1$
Rate = $(1.1 - 1) \times 100 = 10\%$
Answer: 10% per year

2 Ordering numbers

Write these numbers in order of size, starting with the smallest.

0.73, $\frac{1}{4}$, 56%, 69%

Convert each to a decimal: 0.73 = 0.73, $\frac{1}{4}$ = 0.25, 56% = 0.56, 69% = 0.69

In order: $\frac{1}{4}$, 56%, 69%, 0.73

Answer: $\frac{1}{4}$, 56%, 69%, 0.73

3 Independent events and probability rules

A and B are two events.
P(A) = 0.6, P(B) = 0.4 and P(A or B) = 0.9
(a) Work out P(A and B).
(b) Are A and B independent? You must show your working.

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$0.9 = 0.6 + 0.4 - P(A \cap B)$, so $P(A \cap B) = 0.1$
(b) $P(A) \times P(B) = 0.6 \times 0.4 = 0.24$
This is not equal to 0.1, so A and B are not independent
Answer: (a) 0.1 (b) No

4 Quadratic formula and surds

Solve $3x^2 + 7x - 5 = 0$ (Calculator)
Give your answers correct to 2 decimal places.

Use the quadratic formula with a = 3, b = 7, c = −5

$$x = \frac{-7 \pm \sqrt{7^2 - 4 \times 3 \times (-5)}}{2 \times 3} = \frac{-7 \pm \sqrt{109}}{6}$$

x = −2.9067... or x = 0.5734...
x = −2.91 or x = 0.57 (2 d.p.)
Answer: x = −2.91 or x = 0.57

5 Sine rule

The diagram shows triangle ABC. (Calculator)
Angle BAC = 66° and angle ABC is acute.
Work out the size of angle ABC, marked θ. Give your answer correct to 1 decimal place.
The diagram is not drawn accurately.

Use the sine rule: $\frac{\sin\theta}{AC} = \frac{\sin A}{BC}$

$$\frac{\sin\theta}{11} = \frac{\sin 66°}{17}$$

$$\sin\theta = \frac{11 \times \sin 66°}{17} = 0.59112$$

$\theta = \sin^{-1}(0.59112) = 36.2°$ (1 d.p.)
Answer: $\theta = 36.2°$

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6 Fraction problems

A bath is $\frac{3}{5}$ full.

40 more litres of water are added and the bath is then $\frac{4}{5}$ full.

Work out the capacity of the bath.

Fraction added = $\frac{4}{5} - \frac{3}{5} = \frac{1}{5}$

$\frac{1}{5}$ of the capacity is 40 litres

Capacity = $40 \div \frac{1}{5} = 200$

Answer: 200 litres

7 Equal roots and real roots

The equation $x^2 + 8x + k = 0$ has two different real solutions.
Find the range of possible values of k.

Two different real solutions means $b^2 - 4ac > 0$
$8^2 - 4 \times 1 \times k > 0$, so $64 - 4k > 0$
$64 > 4k$, so $k < 16$
Answer: $k < 16$

8 Moving averages

The table shows the daily visitors over six periods.
The 3-point moving average centred on Thu is 24.
(a) Work out the value of x.
(b) Work out the 3-point moving average centred on Fri.

(a) $(26 + x + 19) \div 3 = 24$
$26 + x + 19 = 72$, so $x = 27$
(b) $(27 + 19 + 11) \div 3 = 57 \div 3 = 19$
Answer: (a) x = 27 (b) 19

9 Loci and regions

ABCD is a rectangle with AB = 12 cm and AD = 13 cm.
A point P inside the rectangle is such that AP ≤ 12 cm and P is nearer to A than to B.
Work out the area of the region in which P can lie. Give your answer in terms of π and surds.

Nearer to A than to B: P is on the A side of the perpendicular bisector of AB, a distance 6 cm from A
AP ≤ 12 cm: P is inside the quarter circle of radius 12 cm centred at A

The bisector meets the arc where $\cos\theta = \frac{6}{12}$, so $\theta = 60°$ at A. The region is a right-angled triangle plus a sector of angle 30°

Triangle: base 6 cm, height $12 \sin 60° = 6\sqrt{3}$ cm, area = $\frac{1}{2} \times 6 \times 6\sqrt{3} = 18\sqrt{3}$

Sector of angle 30° : $\frac{30}{360} \times \pi \times 12^2 = 12\pi$

Area = $18\sqrt{3} + 12\pi$ cm^2
Answer: $(18\sqrt{3} + 12\pi)$ cm^2

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10 Algebraic fractions: multiplying and dividing

Simplify fully $\frac{x^2 + 8x + 15}{x^2 - 7x} \times \frac{x^2 + 6x}{x + 3}$

Factorise: $\frac{(x + 3)(x + 5)}{x(x - 7)} \times \frac{x(x + 6)}{(x + 3)}$

Cancel (x + 3) and x

$= \frac{(x + 5)(x + 6)}{(x - 7)}$

Answer: $\mathbf{\frac{(x + 5)(x + 6)}{(x - 7)}}$