Day 329 **S Tier 10-a-day: Model Solutions**

1 Fractions removed from a ratio

The ratio of blue to green sweets in a tin is 4 : 3.

Then $\frac{2}{3}$ of the blue and $\frac{1}{5}$ of the green are taken out.

Work out the ratio of blue to green now.

Left: $\frac{1}{3} \times 4 = \frac{4}{3}$ of the blue part, $\frac{4}{5} \times 3 = \frac{12}{5}$ of the green part

$\frac{4}{3} : \frac{12}{5}$

Multiply by 15: 20 : 36 = 5 : 9

Answer: 5 : 9

2 Perpendicular lines

The diagram shows the straight line L, which passes through the points A and B.
The line M is perpendicular to L and passes through B.
Find the equation of M in the form y = mx + c.

Gradient of L = $\frac{8 - 0}{0 - (-2)} = 4$

Perpendicular gradient = $-1 \div 4 = -\frac{1}{4}$

M passes through B(−2, 0): $0 = (-\frac{1}{4}) \times (-2) + c$, so $c = -\frac{1}{2}$

$y = -\frac{1}{4}x - \frac{1}{2}$

Answer: $y = -\frac{1}{4}x - \frac{1}{2}$

3 Density of an alloy

An alloy is made by mixing 60 cm^3 of tin with 40 cm^3 of iron.
The density of tin is 7.3 g/cm^3. The density of iron is 7.9 g/cm^3.
Work out the density of the alloy.

Mass of tin = 60 × 7.3 = 438 g
Mass of iron = 40 × 7.9 = 316 g
Total mass = 754 g, total volume = 100 cm^3
Density = 754 ÷ 100 = 7.54 g/cm^3
Answer: 7.54 g/cm^3

4 Simultaneous equations

Solve the simultaneous equations
−4x + 5y = 30
−2x + 5y = 20

To match the x terms: (2) × 2 gives −4x + 10y = 40
Subtract the equations to eliminate x: −5y = −10
y = 2
Substitute y = 2 into (1): −4x + 10 = 30
−4x = 20, so x = −5
Answer: x = −5, y = 2

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5 Recurring decimals

Convert the recurring decimal $0.\dot{8}\dot{2}$ to a fraction.
Give your fraction in its simplest form.

Let x = 0.8282...
100x = 82.8282...
Subtract: 99x = 82
$x = \frac{82}{99}$

Answer: $\frac{82}{99}$

6 Sine rule

The diagram shows triangle ABC. (Calculator)
Angle BAC = 33° and angle ABC is acute.
Work out the size of angle ABC, marked θ. Give your answer correct to 1 decimal place.
The diagram is not drawn accurately.

Use the sine rule: $\frac{\sin\theta}{AC} = \frac{\sin A}{BC}$

$\frac{\sin\theta}{15} = \frac{\sin 33°}{16}$

$\sin\theta = \frac{15 \times \sin 33°}{16} = 0.51060$

$\theta = \sin^{-1}(0.51060) = 30.7°$ (1 d.p.)

Answer: $\boldsymbol{\theta = 30.7°}$

7 Profit and loss

A retailer buys a watch and adds a 50% mark-up to get the selling price.
In a sale the selling price is reduced by 15% to £395.25.
Work out how much the shop paid for a watch.

Overall multiplier = 1.5 × 0.85 = 1.275
Cost = £395.25 ÷ 1.275
= £310

Answer: £310

8 Exact trigonometric ratios from a given ratio

x is an obtuse angle and $\sin x = \frac{8}{9}$.

(a) Find the exact value of cos x.
(b) Find the exact value of tan x. Give each answer as a fraction or a surd. Do not use a calculator.

(a) $\cos^2 x = 1 - (\frac{8}{9})^2 = \frac{17}{81}$

x is obtuse, so cos x is negative: $\cos x = -\frac{\sqrt{17}}{9}$

(b) $\tan x = \frac{\sin x}{\cos x} = \frac{8}{9} \div (-\frac{\sqrt{17}}{9}) = -\frac{8\sqrt{17}}{17}$

Answer: (a) $\boldsymbol{-\frac{\sqrt{17}}{9}}$ **(b)** $\boldsymbol{-\frac{8\sqrt{17}}{17}}$

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9 Ratio and money transferred

Dina and Oscar share some money in the ratio 9 : 7.
Dina gives Oscar £9.
The ratio of their money is now 9 : 8.
How much money do they have altogether?

Let the amounts be 9k and 7k, so the total is 16k

Dina now has $9k - 9$, which is $\frac{9}{17}$ of the total

$17(9k - 9) = 9 \times 16k$
$153k - 153 = 144k$, so $9k = 153$ and $k = 17$
Total = 16×17 = £272
Answer: £272

10 Tangency conditions

The line $y = kx - 4$ is a tangent to the curve $y = x^2 + 4$.
Find the possible values of k.
Give any surds in their simplest form.

Equate: $x^2 + 4 = kx - 4$
Rearrange: $x^2 - kx + 8 = 0$
One point of contact, so $b^2 - 4ac = 0$: $k^2 - 4 \times 8 = 0$
$k^2 = 32$
$k = \pm\sqrt{32} = \pm 4\sqrt{2}$
Answer: $k = \pm 4\sqrt{2}$