Day 328 **S Tier 10-a-day: Model Solutions**

1 Similar triangles with algebra

In the diagram, DE is parallel to BC.
AD = x cm, DB = (x + 4) cm, DE = 8 cm and BC = 20 cm.
(a) Find the value of x.
(b) Work out the length of AB.
The diagram is not drawn accurately.

(a) Triangles ADE and ABC are similar, so $\frac{AD}{AB} = \frac{DE}{BC}$

$\frac{x}{2x + 4} = \frac{8}{20}$
$20x = 8(2x + 4)$
$20x = 16x + 32$
$4x = 32$
$x = 8$
(b) AB = 2x + 4 = 20 cm
Answer: (a) x = 8 (b) 20 cm

2 Equation of a circle

A and B are the points (7, −5) and (1, 3).
AB is a diameter of a circle.
Find an equation of the circle.

The centre is the midpoint of AB: $(\frac{7 + 1}{2}, \frac{-5 + 3}{2}) = (4, -1)$
$r^2 = (7 - 4)^2 + (-5 - (-1))^2 = 9 + 16 = 25$
$(x - 4)^2 + (y + 1)^2 = 25$
Answer: $(x - 4)^2 + (y + 1)^2 = 25$

3 Multiplying and dividing surds

Simplify fully $\frac{4\sqrt{150}}{2\sqrt{6}}$

$\sqrt{150} = 5\sqrt{6}$
$\frac{4\sqrt{150}}{2\sqrt{6}} = \frac{20\sqrt{6}}{2\sqrt{6}}$
$= 10$
Answer: 10

4 Averages with algebra

Here are five numbers: 2x − 3, x + 3, 2x + 3, 3x − 3, 2x.
The mean of the five numbers is 16.
(a) Work out the value of x.
(b) Work out the range of the five numbers.

(a) Total = (2x − 3) + (x + 3) + (2x + 3) + (3x − 3) + (2x) = 16 × 5 = 80
10x + 0 = 80, so 10x = 80 and x = 8
The numbers are 13, 11, 19, 21, 16
(b) Range = 21 − 11 = 10
Answer: (a) x = 8 (b) 10

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5 Quadratic sequences

Here are the first five terms of a quadratic sequence.
19, 34, 53, 76, 103
Find an expression for the nth term.

First differences: 15, 19, 23, 27
Second differences: 4, 4, 4, so a = 4 ÷ 2 = 2
Subtract $2n^2$ from each term: 17, 26, 35, 44, 53
The remaining linear sequence has nth term 9n + 8
nth term = $2n^2 + 9n + 8$
Answer: $2n^2 + 9n + 8$

6 Cosine rule and area of a triangle

The diagram shows triangle ABC. (Calculator)
(a) Work out angle ACB, marked θ, to 1 decimal place.
(b) Work out the area of triangle ABC to 3 significant figures.
The diagram is not drawn accurately.

(a) $\cos\theta = \frac{5^2 + 8^2 - 6^2}{2 \times 5 \times 8} = \frac{53}{80}$
θ = 48.5° (1 d.p.)
(b) Area = $\frac{1}{2} \times 5 \times 8 \times \sin 48.5°$
= 15.0 cm^2 (3 s.f.)
Answer: (a) 48.5° (b) 15.0 cm^2

7 Line of best fit

The scatter graph shows age of car (x) and value (y).
The line of best fit passes through (4, 15) and (8, 13).
(a) Find its equation in the form y = mx + c.
(b) Explain what its gradient tells you.

(a) Gradient m = $\frac{13 - 15}{8 - 4} = \frac{-2}{4} = -0.5$
y = −0.5x + c: using (4, 15), 15 = −0.5 × 4 + c, so c = 17
y = −0.5x + 17
(b) For each extra year of age, the value of the car (in £100s) goes down by 0.5
Answer: (a) y = −0.5x + 17 (b) For each extra year of age, the value of the car (in £100s) goes down by 0.5

8 Surds with unknowns

Given that $(a + \sqrt{2})^2 = 83 + 18\sqrt{2}$,
where a is a positive integer, find the value of a.

$(a + \sqrt{2})^2 = a^2 + 2a\sqrt{2} + 2$
Compare the $\sqrt{2}$ terms: 2a = 18, so a = 9
Check the integer terms: $a^2 + 2 = 81 + 2 = 83$
Answer: a = 9

9 Joint and inverse variation

P is directly proportional to the square of v and inversely proportional to the square of t.
When v = 3 and t = 2, P = 9.
Work out the value of P when v = 5 and t = 1.

$P = \frac{kv^2}{t^2}$
$9 = \frac{k \times 9}{4}$, so $k = 9 \times 4 \div 9 = 4$
$P = \frac{4 \times 25}{1} = 100$
Answer: P = 100

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10 Upper and lower bounds

a = 9.6 and b = 3.8,
both correct to 1 decimal place.
Work out the upper bound of a ÷ b.
Give your answer correct to 3 significant figures.

Each value is rounded, so it can be up to 0.05 above or below
$9.55 \leq a < 9.65$; $3.75 \leq b < 3.85$
Biggest answer: biggest a ÷ smallest b = 9.65 ÷ 3.75
= 2.57333... = 2.57 to 3 significant figures
Answer: 2.57