Day 327 **S Tier 10-a-day: Model Solutions**

1 Standard form

Work out $(1.9 \times 10^{-4}) \times (6.4 \times 10^{-2})$
Give your answer in standard form.

$1.9 \times 6.4 = 12.16$
$10^{-4} \times 10^{-2} = 10^{-6}$
$12.16 \times 10^{-6} = 1.216 \times 10^{-5}$
Answer: 1.216×10^{-5}

2 Mean of a changing set

The mean of 8 numbers is 73.
One number is removed and the mean of the remaining 7 numbers is 75.
Work out the number that was removed.

Total of the 8 numbers = $8 \times 73 = 584$
Total of the 7 remaining = $7 \times 75 = 525$
Number removed = $584 - 525 = 59$
Answer: 59

3 Exact trigonometric values

Work out the exact value of $(\cos 60°)^2 + \cos 45°$
Give your answer in its simplest form, as a fraction or surd. Do not use a calculator.

$(\cos 60°)^2 + \cos 45° = (\frac{1}{2})^2 + \frac{\sqrt{2}}{2}$
$= \frac{1}{4} + \frac{\sqrt{2}}{2}$
Answer: $\frac{1}{4} + \frac{\sqrt{2}}{2}$

4 Quadratic formula and surds

Solve $x^2 - 12x + 3 = 0$
Give your answers in the form $a \pm \sqrt{b}$

Complete the square: $(x - 6)^2 - 36 + 3 = 0$
$(x - 6)^2 = 33$
$x - 6 = \pm \sqrt{33}$
$x = 6 \pm \sqrt{33}$
Answer: $x = 6 \pm \sqrt{33}$

5 Column vectors

$\mathbf{a} = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -4 \\ -5 \end{pmatrix}$
Work out the exact length of the vector $\mathbf{a} + \mathbf{b}$. Give your answer as a simplified surd.

$\mathbf{a} + \mathbf{b} = \begin{pmatrix} -1 \\ -1 \end{pmatrix} + \begin{pmatrix} -4 \\ -5 \end{pmatrix} = \begin{pmatrix} -5 \\ -6 \end{pmatrix}$
Length $= \sqrt{(-5)^2 + (-6)^2} = \sqrt{61}$
$= \sqrt{61}$
Answer: $\sqrt{61}$

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6 Using a line of best fit

The scatter graph shows hours of revision (x) and test mark (y).
The equation of the line of best fit is y = 3x + 7.
(a) Estimate y when x = 9.
(b) Explain why an estimate of y when x = 25 may be unreliable.

(a) $y = 3 \times 9 + 7 = 34$
(b) x = 25 is outside the range of the data on the graph, so the estimate relies on the pattern continuing (extrapolation)
Answer: (a) 34 (b) x = 25 is outside the range of the data

7 Volume and surface area of composite solids

A solid is made from a cone of height 5 cm and a hemisphere. Both have radius 3 cm.
Work out the total volume of the solid. Give your answer in terms of π.

Cone: $\frac{1}{3} \times \pi \times 3^2 \times 5 = 15\pi$

Hemisphere: $\frac{2}{3} \times \pi \times 3^3 = 18\pi$

Total = $15\pi + 18\pi = 33\pi$ cm^3
Answer: 33π cm^3

8 Reverse percentages

The price of a watch is increased by 20%.
Later the new price is reduced by 25% in a sale.
The sale price is £72.
Work out the price before the increase. (Calculator)

Multipliers: 1.2 and 0.75
Overall multiplier = 1.2 × 0.75 = 0.9
Original × 0.9 = £72
Original = £72 ÷ 0.9 = £80
Answer: £80

9 Recurring decimals

Without using a calculator, work out $0.\dot{2}\dot{4} + 0.0\dot{6}$
Give your answer as a fraction in its simplest form.

$0.\dot{2}\dot{4} = \frac{8}{33}$

$0.0\dot{6} = \frac{1}{15}$

$\frac{8}{33} + \frac{1}{15} = \frac{17}{55}$

Answer: $\frac{17}{55}$

10 Terms of a quadratic sequence

The nth term of a sequence is $2n^2 + 4n + 2$.
(a) Work out the 12th term.
(b) Is 128 a term of the sequence? You must show your working.

(a) $2 \times 12^2 + 4 \times 12 + 2 = 338$
(b) Solve $2n^2 + 4n + 2 = 128$, so $2n^2 + 4n - 126 = 0$
Factorise or use the formula: n = 7 or n = −9
n = 7 is a positive whole number, so 128 is term number 7
Answer: (a) 338 (b) Yes, it is the 7th term