Day 326 **S Tier 10-a-day: Model Solutions**

1 Time zones

A flight leaves London at 06:35 (London time).
It takes 12 hours 35 minutes.
When it is 12:00 in London it is 19:00 in Singapore.
At what time, in Singapore time, does the flight land? Say if it is on a different day.

Arrival in London time = 06:35 + 12 h 35 min = 19:10
Singapore is 7 hours ahead of London, so add 7 hours
02:10 the next day
Answer: 02:10 the next day

2 Sine rule

The diagram shows triangle ABC. (Calculator)
Angle BAC = 42° and angle ABC is acute.
Work out the size of angle ABC, marked θ. Give your answer correct to 1 decimal place.
The diagram is not drawn accurately.

Use the sine rule: $\frac{\sin\theta}{AC} = \frac{\sin A}{BC}$

$\frac{\sin\theta}{6} = \frac{\sin 42°}{15}$

$\sin\theta = \frac{6 \times \sin 42°}{15} = 0.26765$

$\theta = \sin^{-1}(0.26765) = 15.5°$ (1 d.p.)
Answer: $\theta = 15.5°$

3 Rationalising the denominator

Rationalise the denominator of $\frac{16}{4\sqrt{5}}$
Give your answer in its simplest form.

Multiply top and bottom by $\sqrt{5}$:

$\frac{16}{4\sqrt{5}} = \frac{16\sqrt{5}}{20}$

$= \frac{4\sqrt{5}}{5}$

Answer: $\frac{4\sqrt{5}}{5}$

4 Pressure

A stone block in the shape of a cuboid weighs 330 N.
Its smallest face has area 100 cm^2. Its largest face has area 2,500 cm^2.
Pressure = force ÷ area
Work out (a) the greatest pressure and (b) the least pressure the stone block can exert on the ground, in N/m^2.

1 m^2 = 10 000 cm^2, so 100 cm^2 = 0.01 m^2 and 2,500 cm^2 = 0.25 m^2
(a) Greatest pressure is on the smallest face: 330 ÷ 0.01 = 33,000 N/m^2
(b) Least pressure is on the largest face: 330 ÷ 0.25 = 1,320 N/m^2
Answer: (a) 33,000 N/m^2 (b) 1,320 N/m^2

5 Equations with indices

Solve $x^{\frac{3}{4}} = 8$

Write 8 as a power of 2: $8 = 2^3$
$x^{\frac{3}{4}} = 2^3$, so $x = (2^3)^{\frac{4}{3}}$
$x = 2^4 = 16$
Answer: x = 16

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6 Tangents and circle theorems

PA and PB are tangents to a circle, centre O. C is a point on the circle. Angle APB = 46°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) Angles OAP and OBP are 90° (a tangent meets a radius at 90°)
Angles in quadrilateral OAPB: 360 − 90 − 90 − 46 = 134°
(b) The angle at the circumference is half the angle at the centre: 134 ÷ 2 = 67°
Answer: (a) 134° (b) 67°

7 Rearranging when the subject appears twice

Make u the subject of $v = \frac{2u + 7}{u - 5}$

Multiply by u − 5: v(u − 5) = 2u + 7
Expand: uv − 5v = 2u + 7
Collect the u terms: uv − 2u = 5v + 7
Factorise: u(v − 2) = 5v + 7
$u = \frac{5v + 7}{v - 2}$

Answer: u = $\frac{5v + 7}{v - 2}$

8 Perpendicular bisector

A is the point (−5, 8) and B is the point (1, 4).
Find an equation of the perpendicular bisector of AB.
Give your answer in the form ax + by = c, where a, b and c are integers.

Midpoint of AB = $(\frac{-5 + 1}{2}, \frac{8 + 4}{2}) = (-2, 6)$

Gradient of AB = $\frac{4 - 8}{1 - (-5)} = -\frac{2}{3}$

The perpendicular gradient is $-1 \div (-\frac{2}{3}) = \frac{3}{2}$

Using the midpoint: $y - 6 = \frac{3}{2}(x + 2)$

Multiply by 2 and rearrange: −3x + 2y = 18
Multiply by −1: 3x − 2y = −18
Answer: 3x − 2y = −18

9 Winning a match

Amir and Nadia play a match. The first to win 2 table tennis games wins the match.
The probability that Amir wins any one of the table tennis games is $\frac{3}{4}$.
Work out the probability that the match needs a third one to decide it.

A third one is needed when each player wins one of the first two

P(Amir wins then loses) = $\frac{3}{4} \times \frac{1}{4} = \frac{3}{16}$

P(Amir loses then wins) = $\frac{1}{4} \times \frac{3}{4} = \frac{3}{16}$

Total = $2 \times \frac{3}{16} = \frac{3}{8}$

Answer: $\frac{3}{8}$

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10 Volume and surface area of composite solids

A solid is made from a cone of height 8 cm and a hemisphere. Both have radius 9 cm.
Work out the total volume of the solid. Give your answer in terms of π.

Cone: $\frac{1}{3} \times \pi \times 9^2 \times 8 = 216\pi$

Hemisphere: $\frac{2}{3} \times \pi \times 9^3 = 486\pi$

Total = $216\pi + 486\pi = 702\pi$ cm^3

Answer: 702π cm^3