Day 325 **S Tier 10-a-day: Model Solutions**

1 Joint and inverse variation

F is directly proportional to m and inversely proportional to the square of d.
When m = 5 and d = 1, F = 40.
Work out the value of F when m = 1 and d = 2.

$F = \frac{km}{d^2}$

$40 = \frac{k \times 5}{1}$, so $k = 40 \times 1 \div 5 = 8$

$F = \frac{8 \times 1}{4} = 2$

Answer: F = 2

2 Angles with algebra

In triangle PQR, side QR is extended to S.
Angle QPR = (3x + 15)°
Angle PQR = (2x)°
Angle PRS = (4x + 39)°
(a) Work out the value of x.
(b) Work out the size of angle PRQ.

(a) The exterior angle of a triangle equals the sum of the two opposite interior angles
4x + 39 = 5x + 15
x = 24
(b) Angle PRS = 135° so angle PRQ = 180° − 135° = 45°
Answer: (a) x = 24 (b) 45°

3 Rational and irrational numbers

Here are six numbers.

$0.\dot{4}$, $\frac{\sqrt{20}}{\sqrt{5}}$, $\sqrt[3]{27}$, $(1+\sqrt{2})^2$, $\sqrt{3} \times \sqrt{5}$, $\sqrt{8} \times \sqrt{2}$

Write down the irrational numbers.

Simplify each number:

$0.\dot{4} = \frac{4}{9}$; $\frac{\sqrt{20}}{\sqrt{5}} = \sqrt{4} = 2$; $\sqrt[3]{27} = 3$

$(1+\sqrt{2})^2 = 3 + 2\sqrt{2}$; $\sqrt{3} \times \sqrt{5} = \sqrt{15}$; $\sqrt{8} \times \sqrt{2} = \sqrt{16} = 4$

Answer: $(1+\sqrt{2})^2$, $\sqrt{3} \times \sqrt{5}$

4 Iteration

Use the iteration formula $x_{n+1} = 3 + \frac{5}{x_n}$ with $x_0 = 3$ (Calculator)

to work out x_1, x_2 and x_3.
Give each answer correct to 3 decimal places.

$x_1 = 3 + \frac{5}{3} = 4.66667...$

$x_2 = 3 + \frac{5}{x_1} = 4.07143...$

$x_3 = 3 + \frac{5}{x_2} = 4.22807...$

Rounded: $x_1 = 4.667$, $x_2 = 4.071$, $x_3 = 4.228$

Answer: $x_1 = 4.667$, $x_2 = 4.071$, $x_3 = 4.228$

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5 Percentage change

The price of a phone is increased by 60%.
The new price is then increased by 15%.
Work out the overall percentage change in the price.

Multipliers: 1.6 and 1.15
Overall multiplier = 1.6 × 1.15 = 1.84
1.84 − 1 = 0.84, so an overall increase of 84%
Answer: 84% increase

6 Speed-time graphs

The graph shows the speed of a cyclist during 14 seconds.
(a) Work out the acceleration during the first 4 seconds.
(b) Work out the average speed over the whole 14 seconds.

(a) Acceleration = change in speed ÷ time = (12 − 4) ÷ 4 = 2 m/s^2
(b) Distance = area under the graph
Trapezium: $\frac{4+12}{2} \times 4 = 32$; rectangle: 12 × 8 = 96; triangle: $\frac{1}{2} \times 2 \times 12 = 12$
Total = 140 m, so average speed = 140 ÷ 14 = 10 m/s
Answer: (a) 2 m/s^2 (b) 10 m/s

7 Hollow cylinder and density

A metal pipe is a hollow cylinder with outer radius 5 cm and inner radius 3 cm. (Calculator)
The density of the metal is 7.3 g/cm^3.
The mass of the pipe is 29.4 kg.
Work out the length of the pipe. Give your answer in centimetres correct to 3 significant figures.

Mass = 29400 g, so volume = mass ÷ density = 29400 ÷ 7.3 = 4027.4 cm^3
Cross-sectional area = $\pi \times 5^2 - \pi \times 3^2$ = 50.265 cm^2
Length = volume ÷ area = 4027.4 ÷ 50.265 = 80.1 cm (3 s.f.)
Answer: 80.1 cm

8 Exponential graphs

The diagram shows the graph of $y = ka^x$, where k and a are constants and a > 0.
The curve passes through the points (1, 6) and (4, 162).
Find the value of k and the value of a.

Substitute the points: $k\,a^1 = 6$ and $k\,a^4 = 162$
Divide: $a^3 = \frac{162}{6} = 27$
a = 3
From the first point: k × 3 = 6, so k = 2
Answer: k = 2, a = 3

9 Combining means

The mean of the heights in cm for the men is 69.
The mean of the heights in cm for the women is 76.
The mean for all of the men and women together is 71.
Work out the ratio of the number of men to the number of women.

Let there be p men and q women
Total = 69p + 76q, and the overall mean gives 69p + 76q = 71(p + q)
−2p = −5q
p : q = −5 : −2 = 5 : 2
Answer: 5 : 2

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10 Functions that are their own inverse

$f(x) = \frac{-2x - 3}{2x + 2}$ where $x \neq -1$

(a) Show that $ff(x) = x$.

(b) Write down $f^{-1}(x)$.

(a) Replace x in f(x) by f(x), then multiply the top and the bottom by (2x + 2)

$ff(x) = \frac{(-2)(-2x - 3) + (-3)(2x + 2)}{2(-2x - 3) - (-2)(2x + 2)}$

Top $= 4x + 6 - 6x - 6 = -2x$

Bottom $= -4x - 6 + 4x + 4 = -2$

$ff(x) = \frac{-2x}{-2} = x$

(b) ff(x) = x, so f is its own inverse: $f^{-1}(x) = \frac{-2x - 3}{2x + 2}$

Answer: (b) $\mathbf{f^{-1}(x) = \frac{-2x - 3}{2x + 2}}$