Day 324 **S Tier 10-a-day: Model Solutions**

1 Circle and line intersections

Solve the simultaneous equations

$x^2 + y^2 = 85$

$y = -3x - 15$

Substitute into the first equation: $x^2 + (-3x - 15)^2 = 85$

Expand: $10x^2 + 90x + 140 = 0$

Divide by 10: $x^2 + 9x + 14 = 0$

Factorise: $(x + 7)(x + 2) = 0$, so $x = -7$ or $x = -2$

$y = -3x - 15$ gives $y = 6$ and $y = -9$

Answer: $x = -7$, $y = 6$ and $x = -2$, $y = -9$

2 Stratified sampling

The table shows the number of members in each of four groups.

(a) A stratified sample of 22 is taken. Work out how many come from Group C.

(b) A different stratified sample has 6 from Group A. Work out the size of this sample.

Total = 75 + 175 + 175 + 125 = 550

(a) Group C: $\frac{175}{550} \times 22 = 7$

(b) Fraction sampled = $6 \div 75 = \frac{2}{25}$, so sample size = $\frac{2}{25} \times 550 = 44$

Answer: (a) 7 (b) 44

3 Standard form

Work out $(4.2 \times 10^8) \div (1.5 \times 10^{-8})$

Give your answer in standard form.

$4.2 \div 1.5 = 2.8$

$10^8 \div 10^{-8} = 10^{16}$

$= 2.8 \times 10^{16}$

Answer: 2.8×10^{16}

4 Fractions removed from a ratio

The ratio of men to women people at a meeting is 5 : 2.

Then $\frac{2}{5}$ of the men and $\frac{1}{3}$ of the women are taken out.

Work out the ratio of men to women now.

Left: $\frac{3}{5} \times 5 = 3$ of the men part, $\frac{2}{3} \times 2 = \frac{4}{3}$ of the women part

$3 : \frac{4}{3}$

Multiply by 3: 9 : 4

Answer: 9 : 4

5 Exact trigonometry in triangles

ABC is a right-angled triangle.

Work out the exact value of x. Give your answer in its simplest form.

The diagram is not drawn accurately.

x is the hypotenuse AC and the known side is the adjacent side AB, so use cos 30°

$\cos 30° = \frac{\sqrt{3}}{2}$

$x = 10 \div \cos 30° = 10 \div \frac{\sqrt{3}}{2}$

$x = \frac{20\sqrt{3}}{3}$ (after rationalising the denominator)

Answer: $x = \frac{20\sqrt{3}}{3}$

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6 Percentage change

A price is decreased by 75%.
By what percentage must the new value be increased to get back to the original value?

Let the original be 100. After a 75% decrease it is 25.
To return to 100, increase by 75
Percentage increase = $\frac{75}{25} \times 100 = 300\%$

Answer: 300%

7 Direct proportion formulae

y is directly proportional to the square root of x.
When x = 1, y = 3.
(a) Find a formula for y in terms of x.
(b) Work out the value of y when x = 4.

(a) $y = k\sqrt{x}$
$3 = k \times 1$, so $k = 3 \div 1 = 3$
$y = 3\sqrt{x}$
(b) $y = 3 \times 2 = 6$
Answer: (a) $y = 3\sqrt{x}$ (b) 6

8 Algebraic fractions: multiplying and dividing

Simplify fully $\frac{x^2 - 8x + 7}{x + 2} \div \frac{x - 7}{x^2 - 3x - 10}$

Factorise: $\frac{(x - 1)(x - 7)}{(x + 2)} \div \frac{(x - 7)}{(x + 2)(x - 5)}$

Turn the second fraction upside down and multiply: $\frac{(x - 1)(x - 7)}{(x + 2)} \times \frac{(x + 2)(x - 5)}{(x - 7)}$

Cancel $(x - 7)$ and $(x + 2)$
$= (x - 5)(x - 1)$
Answer: $(x - 5)(x - 1)$

9 Vector proof: collinear points

OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
OP = $-3\mathbf{a} + 4\mathbf{b}$, OQ = $-4\mathbf{a}$ and OR = $-6\mathbf{a} - 8\mathbf{b}$.
(a) Show that P, Q and R lie on a straight line.
(b) Work out the ratio PQ : QR in its simplest form.

(a) $PQ = OQ - OP = -\mathbf{a} - 4\mathbf{b}$
$QR = OR - OQ = -2\mathbf{a} - 8\mathbf{b}$
QR = 2 PQ, so PQ and QR are parallel, and they share the point Q
So P, Q and R lie on a straight line
(b) PQ : QR = 1 : 2
Answer: (b) 1 : 2

10 Rationalising the denominator

Write $\frac{6 + \sqrt{10}}{7 - \sqrt{10}}$ in the form $p + q\sqrt{10}$
where p and q are rational numbers.

Multiply top and bottom by $(7 + \sqrt{10})$:
Numerator: $(6 + \sqrt{10})(7 + \sqrt{10}) = 52 + 13\sqrt{10}$
Denominator: $49 - 10 = 39$
$= \frac{4 + \sqrt{10}}{3}$

Answer: $\frac{4 + \sqrt{10}}{3}$