Day 323 **S Tier 10-a-day: Model Solutions**

1 Laws of indices

Write $\frac{3^{n+1} \times 9^{n+1}}{9^n}$ in the form 3^{an+c}

where a and c are integers.

$3 = 3^1$, $9 = 3^2$, $9 = 3^2$

$3^{n+1} = 3^{n+1}$, $9^{n+1} = 3^{2(n+1)}$, $9^n = 3^{2n}$

Exponent: $(n + 1) + 2(n + 1) - 2n = n + 3$

Answer: a = 1, c = 3

2 Quadratic inequalities

Solve $x^2 \geq 4x + 5$

Rearrange so that one side is 0: $x^2 - 4x - 5 \geq 0$

Solve $x^2 - 4x - 5 = 0$: $(x + 1)(x - 5) = 0$, so $x = -1$ or $x = 5$

The graph is a U shape crossing the x-axis at -1 and 5; it is above the axis outside the roots

$x \leq -1$ or $x \geq 5$

Answer: $x \leq -1$ or $x \geq 5$

3 Weighted percentages

The ratio of Year 10 students to Year 11 students in a school is 3 : 2.
60% of the Year 10 students and 15% of the Year 11 students have a part-time job.
What percentage of all of these students have a part-time job?

Imagine 100 students in total: 60 Year 10 students and 40 Year 11 students

60% of 60 = 36 and 15% of 40 = 6

Together: 42 out of 100

42 ÷ 100 × 100 = 42%

Answer: 42%

4 HCF and LCM

(a) Write 225 and 1350 as products of prime factors.
(b) Find the HCF and the LCM of 225 and 1350.
Give your answers as numbers.

(a) $225 = 3^2 \times 5^2$ and $1350 = 2 \times 3^3 \times 5^2$

(b) HCF = $3^2 \times 5^2 = 225$

LCM = $2 \times 3^3 \times 5^2 = 1350$

Answer: (a) $3^2 \times 5^2$; $2 \times 3^3 \times 5^2$ (b) HCF = 225, LCM = 1350

5 Column vectors

The vectors $\begin{pmatrix} 2k \\ 24 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ k \end{pmatrix}$ are parallel.

k is a positive number.
Work out the value of k.

Parallel vectors: one is a multiple of the other, so the ratio of the components is the same

$\frac{2k}{3} = \frac{24}{k}$

$2k \times k = 24 \times 3$

$2k^2 = 72$

$k^2 = 36$

k = 6 (k is positive)

Answer: k = 6

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6 Changing the subject

Make x the subject of $y = 2\sqrt{x - 1} + 5$

Subtract 5: $y - 5 = 2\sqrt{x - 1}$

Divide by 2: $\frac{y - 5}{2} = \sqrt{x - 1}$

Square both sides: $\frac{(y - 5)^2}{4} = x - 1$

$x = \frac{(y - 5)^2}{4} + 1$

Answer: $\mathbf{x = \frac{(y - 5)^2}{4} + 1}$

7 Reverse percentage growth and decay

The population of a town grows by 10% each year.
After 4 years the population is 33,674.
Work out the population 4 years ago. (Calculator)

Yearly multiplier = 1.1

Original $\times 1.1^4 = 33{,}674$

Original $= 33{,}674 \div 1.1^4 = 23{,}000$

Answer: 23,000

8 Powers of standard form

Work out $(3.6 \times 10^5)^2$
Give your answer in standard form.

$(3.6 \times 10^5)^2 = 3.6^2 \times (10^5)^2$

$3.6^2 = 12.96$ and $(10^5)^2 = 10^{10}$

$12.96 \times 10^{10} = 1.296 \times 10^{11}$

Answer: $\mathbf{1.296 \times 10^{11}}$

9 Vectors: intersecting lines

OAB is a triangle. OA = **a** and OB = **b**.
M is on OA with OM : MA = 1 : 3. N is on OB with ON : NB = 1 : 4. AN and BM intersect at X.
(a) Find BM in terms of **a** and **b**.
(b) Find OX in terms of **a** and **b**.
The diagram is not drawn accurately.

(a) OM $= \frac{1}{4}\mathbf{a}$, so BM $= -\mathbf{b} + \frac{1}{4}\mathbf{a} = \frac{1}{4}\mathbf{a} - \mathbf{b}$

(b) X is on BM: BX = s BM, so OX = OB + s BM $= \frac{1}{4}s\mathbf{a} + (1 - s)\mathbf{b}$

X is on AN: OX = OA + t AN = OA + t(ON − OA) $= (1 - t)\mathbf{a} + \frac{1}{5}t\mathbf{b}$

Equate: $\frac{1}{4}s = 1 - t$ and $1 - s = \frac{1}{5}t$

Solving: $s = \frac{16}{19}$ and $t = \frac{15}{19}$

OX $= \frac{4}{19}\mathbf{a} + \frac{3}{19}\mathbf{b}$

Answer: (a) $\mathbf{\frac{1}{4}a - b}$ **(b) OX** $= \mathbf{\frac{4}{19}a + \frac{3}{19}b}$

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10 Geometric sequences

The 2nd term of a geometric sequence is 36.
The 5th term of the sequence is 288.
(a) Work out the common ratio.
(b) Work out the first term.

(a) The common ratio r satisfies $r^3 = 288 \div 36 = 8$
$r = \sqrt[3]{8} = 2$
(b) The 2nd term is $a \times r^1$: $a \times 2 = 36$
$a = 36 \div 2 = 18$
Answer: (a) r = 2 (b) 18