Day 322 **S Tier 10-a-day: Model Solutions**

1 Sine rule

The diagram shows triangle ABC. (Calculator)
Work out the length of AC. Give your answer correct to 1 decimal place.
The diagram is not drawn accurately.

Use the sine rule: $\frac{AC}{\sin B} = \frac{BC}{\sin A}$

$\frac{x}{\sin 65°} = \frac{13}{\sin 36°}$

$x = \frac{13 \times \sin 65°}{\sin 36°} = 20.045$

x = 20.0 cm (1 d.p.)

Answer: AC = 20.0 cm

2 Index equations

Solve $2^{4x+2} = 8^{x-3}$

Write both sides as powers of 2: $8 = 2^3$
Equate the powers: $4x + 2 = 3x - 9$
$x = -11$

Answer: x = −11

3 Hollow cylinder and density

A metal pipe is a hollow cylinder with outer radius 5 cm and inner radius 4 cm. (Calculator)
The density of the metal is 2.7 g/cm^3.
The mass of the pipe is 1.53 kg.
Work out the length of the pipe. Give your answer in centimetres correct to 3 significant figures.

Mass = 1530 g, so volume = mass ÷ density = 1530 ÷ 2.7 = 566.67 cm^3
Cross-sectional area = $\pi \times 5^2 - \pi \times 4^2$ = 28.274 cm^2
Length = volume ÷ area = 566.67 ÷ 28.274 = 20.0 cm (3 s.f.)

Answer: 20.0 cm

4 Percentage change

The price of a phone is increased by 50%.
The new price is then decreased by 5%.
Work out the overall percentage change in the price.

Multipliers: 1.5 and 0.95
Overall multiplier = 1.5 × 0.95 = 1.425
1.425 − 1 = 0.425, so an overall increase of 42.5%

Answer: 42.5% increase

5 Quadratic inequalities

Solve $x^2 \geq 6x - 8$

Rearrange so that one side is 0: $x^2 - 6x + 8 \geq 0$
Solve $x^2 - 6x + 8 = 0$: $(x - 2)(x - 4) = 0$, so x = 2 or x = 4
The graph is a U shape crossing the x-axis at 2 and 4; it is above the axis outside the roots
$x \leq 2$ or $x \geq 4$

Answer: $x \leq 2$ or $x \geq 4$

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6 Histograms with an unmarked scale

The histogram shows the masses of parcels delivered by a courier.
20 of them had a mass between 30 and 50 kg.
(a) How many had a mass between 0 and 10 kg?
(b) How many parcels are there in total?

Frequency is proportional to the area of a bar
30 to 50: bar is 1 squares high and 20 wide, area 20, which represents 20
So 1 unit of area represents 20 ÷ 20 = 1
(a) 0 to 10: 6 × 10 = 60 units of area, so 60 × 1 = 60
(b) Total area = 60 + 60 + 20 + 20 = 160, so total = 160 × 1 = 160
Answer: (a) 60 (b) 160

7 Iteration: rearranging equations

(a) Show that the equation $x^2 - x - 5 = 0$ can be rearranged to give $x = 1 + \frac{5}{x}$ (Calculator)

(b) Use the iteration formula $x_{n+1} = 1 + \frac{5}{x_n}$ with $x_0 = 2$ to work out x_1 and x_2. Give each answer correct to 3 decimal places.

(a) Rearrange $x^2 - x - 5 = 0$: $x^2 - x = 5$
Factorise: $x(x - 1) = 5$
Divide by x: $x - 1 = \frac{5}{x}$, so $x = 1 + \frac{5}{x}$

(b) $x_1 = 1 + \frac{5}{2} = 3.50000...$

$x_2 = 1 + \frac{5}{x_1} = 2.42857...$

$x_1 = 3.500$ and $x_2 = 2.429$ (3 d.p.)
Answer: (b) $x_1 = 3.500$, $x_2 = 2.429$

8 Successive percentage change

The population of a town is increased by 30%.
The new amount is then decreased by 5%.
Work out the overall percentage change.

Multipliers: 1.3 and 0.95
1.3 × 0.95 = 1.235
1.235 is an increase of 23.5%
Answer: 23.5% increase

9 Loci and regions

ABCD is a rectangle with AB = 2 cm and AD = 4 cm.
A point P inside the rectangle is such that AP ≤ 2 cm and P is nearer to A than to B.
Work out the area of the region in which P can lie. Give your answer in terms of π and surds.

Nearer to A than to B: P is on the A side of the perpendicular bisector of AB, a distance 1 cm from A
AP ≤ 2 cm: P is inside the quarter circle of radius 2 cm centred at A

The bisector meets the arc where $\cos\theta = \frac{1}{2}$, so $\theta = 60°$ at A. The region is a right-angled triangle plus a sector of angle 30°

Triangle: base 1 cm, height $2 \sin 60° = 1\sqrt{3}$ cm, area $= \frac{1}{2} \times 1 \times 1\sqrt{3} = \frac{1}{2}\sqrt{3}$

Sector of angle 30° : $\frac{30}{360} \times \pi \times 2^2 = \frac{1}{3}\pi$

Area $= \frac{1}{2}\sqrt{3} + \frac{1}{3}\pi$ cm^2

Answer: $(\frac{1}{2}\sqrt{3} + \frac{1}{3}\pi)$ cm^2

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10 Surds and fractions

Simplify $\frac{1}{2+\sqrt{3}} + \frac{1}{2-\sqrt{3}}$

Give your answer with a rational denominator.

Common denominator: $(2+\sqrt{3})(2-\sqrt{3}) = 4 - 3 = 1$

Numerator: $(2-\sqrt{3}) + (2+\sqrt{3})$

$= 4$

$\frac{4}{1} = 4$

Answer: 4