Day 321 **S Tier 10-a-day: Model Solutions**

1 Equations with indices

Solve $x^{-\frac{3}{2}} = \frac{1}{8}$

Write $\frac{1}{8}$ as a power of 2: $\frac{1}{8} = 2^{-3}$

$x^{-\frac{3}{2}} = 2^{-3}$, so $x = (2^{-3})^{-\frac{2}{3}}$

$x = 2^2 = 4$

Answer: x = 4

2 Mean of a changing set

The mean of 10 numbers is 112.
One number is removed and the mean of the remaining 9 numbers is 119.
Work out the number that was removed.

Total of the 10 numbers = 10 × 112 = 1120
Total of the 9 remaining = 9 × 119 = 1071
Number removed = 1120 − 1071 = 49
Answer: 49

3 Solving quadratics by factorising

Solve $4x^2 - 8x - 5 = 0$

Factorise: $(2x + 1)(2x - 5) = 0$
$2x + 1 = 0$ or $2x - 5 = 0$
$x = -\frac{1}{2}$ or $x = \frac{5}{2}$

Answer: $x = -\frac{1}{2}$ or $x = \frac{5}{2}$

4 Probabilities with an unknown

A biased spinner lands on one of four outcomes with these probabilities.
red: 2x; blue: x + 0.2; green: 3x; yellow: 0.2
(a) Work out the value of x.
(b) The biased spinner is used 150 times. Work out an estimate for the number of times it lands on red.

(a) Probabilities add up to 1: $2x + x + 0.2 + 3x + 0.2 = 1$
$6x + 0.4 = 1$, so $6x = 0.6$
$x = 0.1$
(b) P(red) = 2 × 0.1 = 0.2, so 150 × 0.2 = 30
Answer: (a) x = 0.1 (b) 30

5 Changing the subject

Make l the subject of $T = \frac{5}{l^2}$

where l is positive.

Multiply by l^2: $Tl^2 = 5$

Divide by T: $l^2 = \frac{5}{T}$

Square root: $l = \sqrt{\frac{5}{T}}$

Answer: $l = \sqrt{\frac{5}{T}}$

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6 Direct proportion formulae

E is directly proportional to the cube of v.
When v = 4, E = 192.
(a) Find a formula for E in terms of v.
(b) Work out the positive value of v when E = 375.

(a) $E = kv^3$
$192 = k \times 64$, so $k = 192 \div 64 = 3$
$E = 3v^3$
(b) $375 = 3v^3$, so $v^3 = 375 \div 3 = 125$, giving $v = 5$
Answer: (a) $E = 3v^3$ (b) $v = 5$

7 Income tax

In one country, income tax is worked out like this:
The first £12,000 of income is not taxed.
The next £30,000 is taxed at 10%.
All income above that is taxed at 30%.
Hana earns £79,000 in a year.
Work out how much income tax Hana pays.

Taxed at 10%: £30,000
10% of £30,000 = £3,000
Taxed at 30%: £79,000 − £12,000 − £30,000 = £37,000
30% of £37,000 = £11,100
Total tax = £3,000 + £11,100 = £14,100
Answer: £14,100

8 Trigonometric graphs

The diagram shows the graph of $y = a \cos bx$, where a and b are positive integers.
(a) Write down the value of a and the value of b.
(b) Work out the value of y when x = 330°.

(a) The maximum value is 4 and the minimum is −4, so a = 4
The graph completes 3 full cycles between 0° and 360°, so b = 3
(b) $y = 4 \cos (3 \times 330°) = 4 \cos 990°$
$\cos 270° = 0$, so y = 0
Answer: (a) a = 4, b = 3 (b) y = 0

9 Squares, cubes and prime factors

Find the smallest positive integer k such that 1323 × k is a square number.

$1323 = 3^3 \times 7^2$
For a square number every power must be a multiple of 2
Multiply by 3 to get $3^4 \times 7^2$
k = 3
Answer: k = 3

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10 Frustum of a cone

A cone is cut parallel to its base to make a smaller cone and a frustum.
The frustum has height 12 cm, top radius 6 cm and base radius 18 cm.
Work out the volume of the frustum. Give your answer in terms of π.

Let the height of the small cone removed be h cm. The cones are similar, so $\frac{h}{h+12} = \frac{6}{18}$

18 h = 6(h + 12), so 12 h = 72 and h = 6

Large cone: $\frac{1}{3} \times \pi \times 18^2 \times 18 = 1944\pi$

Small cone: $\frac{1}{3} \times \pi \times 6^2 \times 6 = 72\pi$

Frustum = $1944\pi - 72\pi = 1872\pi$ cm^3

Answer: 1872π cm^3