Day 320 **S Tier 10-a-day: Model Solutions**

1 LCM problems

A red light flashes every 5 minutes.
A blue light flashes every 10 minutes.
A green light flashes every 15 minutes.
They all flash together at 8:00 am.
How many times do they all flash together between 8:00 am and 3:00 pm, including both 8:00 am and 3:00 pm if they do?

Find the LCM of 5, 10 and 15
5 = 5, 10 = 2 × 5, 15 = 3 × 5
LCM = 2 × 3 × 5 = 30
Total time from 8:00 am to 3:00 pm = 420 minutes
420 ÷ 30 = 14, so there are 14 more times after the start
Including the start: 14 + 1 = 15
Answer: 15

2 Weighted percentages

The ratio of men to women in a school is 5 : 1.
65% of the men and 50% of the women own a car.
What percentage of all of these students own a car?

Imagine 120 students in total: 100 men and 20 women
65% of 100 = 65 and 50% of 20 = 10
Together: 75 out of 120
75 ÷ 120 × 100 = 62.5%
Answer: 62.5%

3 Tests of divisibility

The 4-digit number 30k0 is a multiple of 6.
k is a single digit.
Find all the possible values of k.

A multiple of 6 is even and a multiple of 3
The last digit is 0, which is even
Digit sum = 3 + 0 + 0 + k = 3 + k
This must be a multiple of 3, so k = 0 or 3 or 6 or 9
Answer: k = 0 or 3 or 6 or 9

4 Combined transformations

The point P(−6, 5) is rotated 90° anticlockwise about the origin.
The image is then reflected in the line y = x.
(a) Write down the coordinates of the final image of P.
(b) Describe fully the single transformation that is equivalent to these two transformations.

(a) After the first transformation: P' = (−5, −6)
After the second transformation: P'' = (−6, −5)
(b) Follow a general point (x, y) through both transformations: (x, y) → (−y, x) → (x, −y)
The single mapping is (x, y) → (x, −y)
This is a reflection in the x-axis
Answer: (a) (−6, −5) (b) Reflection in the x-axis

5 Laws of indices with algebra

Simplify fully $\frac{2x^{\frac{4}{3}} \times 4x^{\frac{3}{4}}}{4x^{\frac{3}{2}}}$

Numbers: 2 × 4 ÷ 4 = 2

Add the powers for × and subtract for ÷: $\frac{4}{3} + \frac{3}{4} - (\frac{3}{2}) = \frac{7}{12}$

$= 2x^{\frac{7}{12}}$

Answer: $2x^{\frac{7}{12}}$

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6 Arc length and sector area

A sector of a circle has angle 216° and area 15π cm^2.
(a) Work out the radius of the circle.
(b) Work out the length of the arc of the sector. Give your answer to (b) in terms of π.

(a) Area = $\frac{216}{360} \times \pi \times r^2$, so $\frac{216}{360} \times r^2 = 15$

$r^2 = 15 \times \frac{360}{216} = 25$

$r = 5$ cm

(b) Arc length = $\frac{216}{360} \times 2 \times \pi \times 5 = 6\pi$ cm

Answer: (a) 5 cm (b) 6π cm

7 Ratio and mixtures

Container A holds juice and water in the ratio 3 : 1.
Container B holds juice and water in the ratio 3 : 3.
12 litres from container A is mixed with some of container B.
The ratio of juice to water in the mixture is 3 : 2.
Work out how many litres were taken from container B.

Let y litres be taken from B

juice: $9 + \frac{3y}{6}$

water: $3 + \frac{3y}{6}$

Ratio 3 : 2, so $2(9 + \frac{3y}{6}) = 3(3 + \frac{3y}{6})$

$(\frac{-1}{2})y = -9$, so $y = 18$

Answer: 18 litres

8 Fractions removed from a ratio

The ratio of red to blue marbles in a jar is 3 : 8.
Then $\frac{2}{3}$ of the red and $\frac{3}{4}$ of the blue are taken out.
Work out the ratio of red to blue now.

Left: $\frac{1}{3} \times 3 = 1$ of the red part, $\frac{1}{4} \times 8 = 2$ of the blue part

1 : 2
Multiply by 1: 1 : 2
Answer: 1 : 2

9 Speed-time graphs

The graph shows the speed of a train between two stations.
(a) Work out the acceleration in the first 4 seconds.
(b) Work out the total distance.

(a) Acceleration = change in speed ÷ time = 4 ÷ 4 = 1 m/s^2
(b) Distance = area under the graph, split into two triangles and a rectangle
$\frac{1}{2} \times 4 \times 4 + 4 \times 10 + \frac{1}{2} \times 4 \times 4 = 8 + 40 + 8$
= 56
Answer: (a) 1 m/s^2 (b) 56 m

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10 Trigonometric graphs

The diagram shows the graph of y = a sin x + c.
(a) Write down the value of a and the value of c.
(b) Work out the value of y when x = 330°.

(a) The maximum value is 4 and the minimum value is −2
a = (4 − (−2)) ÷ 2 = 3
c = (4 + (−2)) ÷ 2 = 1

(b) y = 3 sin 330° + 1, and sin 330° = $-\frac{1}{2}$

y = $-\frac{1}{2}$

Answer: (a) a = 3, c = 1 (b) y = $-\frac{1}{2}$