Day 32 **S Tier 10-a-day: Model Solutions**

1 Weighted percentages

The ratio of men to women in a school is 1 : 5.
55% of the men and 10% of the women own a car.
What percentage of all of these students own a car?

Imagine 120 students in total: 20 men and 100 women
55% of 20 = 11 and 10% of 100 = 10
Together: 21 out of 120
21 ÷ 120 × 100 = 17.5%
Answer: 17.5%

2 Frustum of a cone

A cone is cut parallel to its base to make a smaller cone and a frustum.
The frustum has height 6 cm, top radius 2 cm and base radius 5 cm.
Work out the volume of the frustum. Give your answer in terms of π.

Let the height of the small cone removed be h cm. The cones are similar, so $\frac{h}{h+6} = \frac{2}{5}$

5 h = 2(h + 6), so 3 h = 12 and h = 4

Large cone: $\frac{1}{3} \times \pi \times 5^2 \times 10 = \frac{250}{3}\pi$

Small cone: $\frac{1}{3} \times \pi \times 2^2 \times 4 = \frac{16}{3}\pi$

Frustum = $\frac{250}{3}\pi - \frac{16}{3}\pi = 78\pi$ cm^3

Answer: 78π cm^3

3 Percentage change

The population of a village increased by 5% in 2023.
In 2024 the population increased by a further 10%.
Work out the overall percentage increase over the two years.

Multipliers: 1.05 and 1.1
Overall multiplier = 1.05 × 1.1 = 1.155
1.155 − 1 = 0.155, so an overall increase of 15.5%
Answer: 15.5% increase

4 Areas of similar triangles

In the diagram, DE is parallel to BC. AD = 9 cm, DB = 12 cm and the area of triangle ADE is 180 cm^2.
Work out the area of the trapezium DBCE.
The diagram is not drawn accurately.

Triangles ADE and ABC are similar. AB = 9 + 12 = 21 cm

Length scale factor = 21 ÷ 9 = $\frac{7}{3}$

Area scale factor = $(\frac{7}{3})^2 = \frac{49}{9}$

Area of triangle ABC = 180 × $\frac{49}{9}$ = 980 cm^2

Trapezium = 980 − 180 = 800 cm^2

Answer: 800 cm^2

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5 Quadratic graphs

The curve $y = x^2 - 6x$ crosses the x-axis at the points A and B.
(a) Find the coordinates of A and B.
(b) Find the coordinates of the minimum point of the curve.

(a) Put $y = 0$: $x^2 - 6x = 0$, so $(x)(x - 6) = 0$
$x = 0$ or $x - 6 = 0$, so $x = 0$ or $x = 6$
(b) The turning point is halfway between A and B: $x = (0 + 6) \div 2 = 3$
When $x = 3$: $y = 3^2 - 6 \times 3 + 0 = -9$
Answer: (a) A(0, 0), B(6, 0) (b) (3, −9)

6 Tangent to a circle

A circle has equation $x^2 + y^2 = 841$.
The point P(−21, −20) lies on the circle.
(a) Find the equation of the tangent to the circle at P. Give your answer in the form $ax + by = c$, where a, b and c are integers.
(b) The tangent meets the x-axis at the point T. Write down the coordinates of T.

(a) The gradient of the radius OP is $\frac{20}{21}$

The tangent is perpendicular to the radius, so its gradient is $-1 \div \frac{20}{21} = -\frac{21}{20}$

$y - (-20) = (-\frac{21}{20})(x - (-21))$

Multiply out and rearrange: $21x + 20y = -841$
(b) On the x-axis $y = 0$, so $21x = -841$
$x = -\frac{841}{21}$, so T is $(-\frac{841}{21}, 0)$

Answer: (a) $21x + 20y = -841$ (b) T = $(-\frac{841}{21}, 0)$

7 At least one

The probability that a bus is more than 5 minutes late on any morning is $\frac{1}{10}$.

Events on different occasions are independent.
Work out the probability that this happens at least once in 4 mornings.
Give your answer correct to 3 significant figures. (Calculator)

P(never) = $(1 - \frac{1}{10})^4 = \frac{9}{10}^4 = 0.6561$
P(at least once) = 1 − P(never) = 1 − 0.6561
= 0.344
Answer: 0.344

8 Pythagoras and quadratic equations

The lengths of the sides of a right-angled triangle are (x + 6) cm, (x − 1) cm and (x + 7) cm.
The longest side is (x + 7) cm.
(a) Show that $x^2 - 4x - 12 = 0$
(b) Hence work out the area of the triangle.

(a) Pythagoras: $(x + 6)^2 + (x - 1)^2 = (x + 7)^2$
Expand: $x^2 + 12x + 36 + x^2 - 2x + 1 = x^2 + 14x + 49$
Rearrange: $x^2 - 4x - 12 = 0$
(b) Factorise: $(x - 6)(x + 2) = 0$, so $x = 6$ (the other root, $x = -2$, does not give valid side lengths)
The shorter sides are 12 cm and 5 cm, so area = $\frac{1}{2} \times 12 \times 5 = 30$ cm^2

Answer: (b) 30 cm^2

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9 Ratio and money transferred

Tariq and Kira share some money in the ratio 9 : 5.
Tariq gives Kira £96.
The ratio of their money is now 3 : 7.
How much money do they have altogether?

Let the amounts be 9k and 5k, so the total is 14k

Tariq now has 9k − 96, which is $\frac{3}{10}$ of the total

10(9k − 96) = 3 × 14k
90k − 960 = 42k, so 48k = 960 and k = 20
Total = 14 × 20 = £280
Answer: £280

10 Bounds and accuracy

a = 8.71 and b = 88.6, both correct to 3 significant figures.
Work out a ÷ b.
Write down your answer to a suitable degree of accuracy.
You must show your working and give a reason for your answer. (Calculator)

8.705 ≤ a < 8.715; 88.55 ≤ b < 88.65
Lower bound = 8.705 ÷ 88.65 = 0.098195...
Upper bound = 8.715 ÷ 88.55 = 0.098419...
Both bounds round to 0.098 to 2 significant figures, but not to 3 significant figures
Answer: 0.098 (2 s.f.)