Day 319 **S Tier 10-a-day: Model Solutions**

1 Angles in polygons

The interior angles of a pentagon are
$x°$, $(x - 10)°$, $(x + 10)°$, $(2x - 20)°$, $(x + 20)°$
(a) Work out the value of x.
(b) Work out the size of the largest angle.

(a) Sum of interior angles = $(5 - 2) \times 180 = 540°$
$6x = 540$, so $6x = 540$
$x = 90$
(b) The angles are 90°, 80°, 100°, 160°, 110°
The largest is 160°
Answer: (a) x = 90 (b) 160°

2 Comparing costs

Zip Taxis has a fixed charge of £3.50 plus £4.50 per mile.
Go Cabs has a fixed charge of £45.50 plus £1.50 per mile.
(a) Work out the number of miles for which the two costs are the same.
(b) Which is cheaper for 17 miles, and by how much?

(a) Let n be the number of miles. Working in pounds, the costs are equal when $3.50 + 4.50n = 45.50 + 1.50n$
$3n = 42$, so $n = 14$
(b) Zip Taxis: £3.50 + £4.50 × 17 = £80
Go Cabs: £45.50 + £1.50 × 17 = £71
Go Cabs is cheaper by £9
Answer: (a) 14 (b) Go Cabs, by £9

3 Circle theorems: angle at the centre

A, B and C are points on a circle, centre O. Angle OAB = 45°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) OA = OB (radii), so triangle OAB is isosceles and angle OBA = 45°
Angle AOB = $180 - 2 \times 45 = 90°$
(b) The angle at the circumference is half the angle at the centre: $90 \div 2 = 45°$
Answer: (a) 90° (b) 45°

4 Rational and irrational numbers

Here are six numbers.

$\sqrt{2} + \sqrt{8}$, $(\sqrt{3})^2$, $\frac{\sqrt{20}}{\sqrt{5}}$, $\sqrt{49}$, $2\pi - \pi$, $(\sqrt{6} + 2)(\sqrt{6} - 2)$

Write down the rational numbers.

Simplify each number:

$\sqrt{2} + \sqrt{8} = 3\sqrt{2}$; $(\sqrt{3})^2 = 3$; $\frac{\sqrt{20}}{\sqrt{5}} = \sqrt{4} = 2$

$\sqrt{49} = 7$; $2\pi - \pi = \pi$; $(\sqrt{6} + 2)(\sqrt{6} - 2) = 6 - 4 = 2$

Answer: $(\sqrt{3})^2$, $\frac{\sqrt{20}}{\sqrt{5}}$, $\sqrt{49}$, $(\sqrt{6} + 2)(\sqrt{6} - 2)$

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5 Averages with algebra

Here are five numbers: $x + 6$, $x + 4$, 1, $x - 3$, $2x + 2$.
The mean of the five numbers is 7.
(a) Work out the value of x.
(b) Work out the median of the five numbers.

(a) Total = $(x + 6) + (x + 4) + (1) + (x - 3) + (2x + 2) = 7 \times 5 = 35$
$5x + 10 = 35$, so $5x = 25$ and $x = 5$
The numbers are 11, 9, 1, 2, 12
(b) In order: 1, 2, 9, 11, 12, so the median is 9
Answer: (a) x = 5 (b) 9

6 Perpendicular bisector

A is the point $(2, -4)$ and B is the point $(0, 2)$.
Find an equation of the perpendicular bisector of AB.
Give your answer in the form $ax + by = c$, where a, b and c are integers.

Midpoint of AB = $(\frac{2+0}{2}, \frac{-4+2}{2}) = (1, -1)$

Gradient of AB = $\frac{2-(-4)}{0-2} = -3$

The perpendicular gradient is $-1 \div (-3) = \frac{1}{3}$

Using the midpoint: $y + 1 = \frac{1}{3}(x - 1)$

Multiply by 3 and rearrange: $-x + 3y = -4$
Multiply by -1: $x - 3y = 4$
Answer: $x - 3y = 4$

7 Finding a percentage rate

A car was bought for £1,500.
Each year its value decreases by the same percentage.
After 2 years it is worth £735.
Work out the percentage decrease each year.

Multiplier for 2 years = £735 ÷ £1,500 = 0.49
Yearly multiplier = the square root of 0.49 = 0.7
30% decrease each year
Answer: 30%

8 Algebraic proof (harder)

Prove algebraically that $k^3 + 17k$ is a multiple of 6 for all integer values of k.

$k^3 + 17k = (k^3 - k) + 18k = (k - 1)k(k + 1) + 18k$
$(k - 1)k(k + 1)$ is the product of three consecutive integers, so it contains a multiple of 2 and a multiple of 3, so it is a multiple of 6
$18k = 6 \times 3k$ is also a multiple of 6
The sum of two multiples of 6 is a multiple of 6
Answer: Proved

9 Ratio and expressions

$4x = 3y$
Work out the ratio $(5x + 2y) : (4y - 3x)$
Give your answer in its simplest form.

$4x = 3y$ gives $x : y = 3 : 4$, so let $x = 3t$ and $y = 4t$
$5(3t) + 2(4t) = 23t$
$4(4t) - 3(3t) = 7t$
$23t : 7t = 23 : 7$
Answer: 23 : 7

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10 Index equations

Solve $5^{3x-1} = 25^{x+4}$

Write both sides as powers of 5: $25 = 5^2$
Equate the powers: $3x - 1 = 2x + 8$
$x = 9$
Answer: x = 9