Day 318 **S Tier 10-a-day: Model Solutions**

1 Chords and Pythagoras in circles

AB is a chord of a circle, centre O. M is the midpoint of AB. (Calculator)
(a) Work out the length OM.
(b) Work out angle AOB, to 1 decimal place.
The diagram is not drawn accurately.

(a) AM = 48 ÷ 2 = 24 cm
$OM^2 = OA^2 - AM^2 = 25^2 - 24^2 = 49$
OM = 7 cm

(b) sin(angle AOM) = $\frac{24}{25}$
Angle AOM = 73.74°, so angle AOB = 2 × 73.74 = 147.5°
Answer: (a) 7 cm (b) 147.5°

2 Tests of divisibility

The 4-digit number 696k is a multiple of 4.
k is a single digit.
Find all the possible values of k.

A number is a multiple of 4 if its last two digits form a multiple of 4
Last two digits: 6k
Multiples of 4 of this form: 60, 64, 68
Answer: k = 0 or 4 or 8

3 Pressure

A wooden block in the shape of a cuboid weighs 960 N.
Its smallest face has area 500 cm^2. Its largest face has area 800 cm^2.
Pressure = force ÷ area
Work out (a) the greatest pressure and (b) the least pressure the wooden block can exert on the ground, in N/m^2.

1 m^2 = 10 000 cm^2, so 500 cm^2 = 0.05 m^2 and 800 cm^2 = 0.08 m^2
(a) Greatest pressure is on the smallest face: 960 ÷ 0.05 = 19,200 N/m^2
(b) Least pressure is on the largest face: 960 ÷ 0.08 = 12,000 N/m^2
Answer: (a) 19,200 N/m^2 (b) 12,000 N/m^2

4 Perpendicular bisector

A is the point (5, 0) and B is the point (13, 4).
Find an equation of the perpendicular bisector of AB. Give your answer in the form ax + by = c, where a, b and c are integers.

Midpoint of AB = (9, 2)
Gradient of AB = $\frac{1}{2}$
Perpendicular gradient = −2
y − 2 = (−2)(x − 9)
Rearrange to integer coefficients: 2x + y = 20
Answer: 2x + y = 20

5 Composite functions

f(x) = 5x + 5 and $g(x) = x^2 - 4$
(a) Find gf(x). Give your answer in the form $ax^2 + bx + c$.
(b) Solve gf(x) = 96

(a) $gf(x) = g(5x + 5) = (5x + 5)^2 - 4$
$= 25x^2 + 50x + 21$
(b) $(5x + 5)^2 = 100$, so 5x + 5 = ±10
5x + 5 = 10 or 5x + 5 = −10
x = −3 or x = 1
Answer: (a) $25x^2 + 50x + 21$ (b) x = −3 or x = 1

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6 Weighted percentages

The ratio of boys to girls in a school is 4 : 1.
85% of the boys and 20% of the girls walk to school.
What percentage of all of these students walk to school?

Imagine 100 students in total: 80 boys and 20 girls
85% of 80 = 68 and 20% of 20 = 4
Together: 72 out of 100
72 ÷ 100 × 100 = 72%
Answer: 72%

7 Recurring decimals

Write $0.93\dot{8}$ as a fraction in its simplest form.

Let x = 0.9388...
100x = 93.8888...
1000x = 938.8888...
Subtract: 900x = 845
$x = \frac{845}{900} = \frac{169}{180}$

Answer: $\frac{169}{180}$

8 Equations with algebraic fractions

Solve $\frac{3}{x-2} - \frac{5}{x+2} = 2$

Multiply every term by $(x-2)(x+2)$: $3(x+2) - 5(x-2) = 2 \times (x-2)(x+2)$
Expand and rearrange so that one side is 0: $x^2 + x - 12 = 0$
Factorise: $(x-3)(x+4) = 0$
$x = -4$ or $x = 3$
Answer: $x = -4$ or $x = 3$

9 Percentages with an unknown rate

The price of a share is increased by x%.
The new price is then decreased by x%.
The final price is 4% less than the original price.
Work out the value of x.

Let the original price be 1. Multipliers: $(1 + \frac{x}{100})$ and $(1 - \frac{x}{100})$

$(1 + \frac{x}{100})(1 - \frac{x}{100}) = 1 - \frac{x^2}{10000}$

Final price is 4% less, so the multiplier is 0.96: $1 - \frac{x^2}{10000} = 0.96$

$\frac{x^2}{10000} = 0.04$, so $x^2 = 400$ and $x = 20$

Answer: $x = 20$

10 Mean of a changing set

The mean of 10 numbers is 144.
A new number is added and the mean of all 11 numbers is 133.
Work out the new number.

Total of the 10 numbers = 10 × 144 = 1440
New total = 11 × 133 = 1463
New number = 1463 − 1440 = 23
Answer: 23