Day 317 **S Tier 10-a-day: Model Solutions**

1 Stratified sampling

The table shows the number of students in each of four groups.
(a) A stratified sample of 34 is taken. Work out how many come from Year 8.
(b) A different stratified sample has 22 from Year 7. Work out the size of this sample.

Total = 110 + 90 + 70 + 70 = 340

(a) Year 8: $\frac{90}{340} \times 34 = 9$

(b) Fraction sampled = 22 ÷ 110 = $\frac{1}{5}$, so sample size = $\frac{1}{5} \times 340 = 68$

Answer: (a) 9 (b) 68

2 Angle in a semicircle

AB is a diameter of a circle, centre O. C is a point on the circle. Angle BAC = 26°.
(a) Work out the size of angle ACB.
(b) Work out the size of angle COB.
The diagram is not drawn accurately.

(a) The angle in a semicircle is 90°, so angle ACB = 90°
(b) OA = OC (radii), so triangle OAC is isosceles and angle OCA = 26°
Angle COB is an exterior angle of triangle OAC: 26 + 26 = 52°
(or: the angle at the centre is twice the angle at the circumference)
Answer: (a) 90° (b) 52°

3 Factorising quadratics

Factorise $12x^2 - 41x + 35$

12 × 35 = 420. Two numbers that multiply to 420 and add to −41 are −21 and −20
Split the middle term: $12x^2 - 21x - 20x + 35$
Factorise in pairs: $3x(4x - 7) - 5(4x - 7)$
$= (3x - 5)(4x - 7)$
Answer: $(3x - 5)(4x - 7)$

4 Ratio and algebra

In a bag, the ratio of the number of red counters to the number of blue counters is 3 : 8.
Then 6 more red counters are added and 16 blue counters are removed.
The ratio is now 7 : 8.
How many red counters were there at the start?

Let the numbers at the start be 3k and 8k
(3k + 6) : (8k − 16) = 7 : 8, so 8(3k + 6) = 7(8k − 16)
24k + 48 = 56k − 112
−32k = −160, so k = 5
Number of red counters at the start: 3 × 5 = 15
Answer: 15

5 Enlargement

Triangle T has vertices A(6, −1), B(5, −2), C(1, 1).
T is enlarged with scale factor 3 and centre (3, −2).
(a) Write down the coordinates of the image of A.
(b) The area of T is 12 square units. Work out the area of the image of T.

(a) Vector from the centre to A = (6 − 3, −1 − (−2)) = (3, 1)
Multiply by 3: (9, 3)
Image of A = centre + this vector = (12, 1)
(b) Area scale factor = $(3)^2 = 9$
Area = 12 × 9 = 108 square units
Answer: (a) (12, 1) (b) 108

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6 Powers of standard form

Work out $(1.4 \times 10^{-6})^3$
Give your answer in standard form.

$(1.4 \times 10^{-6})^3 = 1.4^3 \times (10^{-6})^3$
$1.4^3 = 2.744$ and $(10^{-6})^3 = 10^{-18}$
$= 2.744 \times 10^{-18}$
Answer: 2.744×10^{-18}

7 Density of an alloy

An alloy contains 355 g of zinc and 356 g of copper.
The density of zinc is 7.1 g/cm^3. The density of copper is 8.9 g/cm^3.
Work out the density of the alloy.

Volume of zinc = $355 \div 7.1 = 50$ cm^3
Volume of copper = $356 \div 8.9 = 40$ cm^3
Total mass = 711 g, total volume = 90 cm^3
Density = $711 \div 90 = 7.9$ g/cm^3
Answer: 7.9 g/cm^3

8 Tangent to a circle

A circle has equation $x^2 + y^2 = 841$.
The point P$(-21, 20)$ lies on the circle.
(a) Find the equation of the tangent to the circle at P. Give your answer in the form ax + by = c, where a, b and c are integers.
(b) The tangent meets the x-axis at the point T. Write down the coordinates of T.

(a) The gradient of the radius OP is $-\frac{20}{21}$

The tangent is perpendicular to the radius, so its gradient is $-1 \div (-\frac{20}{21}) = \frac{21}{20}$

$y - 20 = \frac{21}{20}(x - (-21))$

Multiply out and rearrange: $21x - 20y = -841$
(b) On the x-axis $y = 0$, so $21x = -841$
$x = -\frac{841}{21}$, so T is $(-\frac{841}{21}, 0)$

Answer: (a) $21x - 20y = -841$ (b) T = $(-\frac{841}{21}, 0)$

9 Squares, cubes and prime factors

Find the smallest positive integer k such that $135 \div k$ is a square number.

$135 = 3^3 \times 5$
For a square number every power must be even
Divide by 3×5 to get 3^2
$k = 15$
Answer: k = 15

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10 Tangency conditions

The line $y = x + k$ is a tangent to the circle $x^2 + y^2 = 17$.
Find the possible values of k.
Give any surds in their simplest form.

Substitute $y = x + k$: $x^2 + (x + k)^2 = 17$
Expand: $2x^2 + 2kx + k^2 - 17 = 0$
A tangent touches at one point, so $b^2 - 4ac = 0$: $(2k)^2 - 4 \times 2 \times (k^2 - 17) = 0$
$-4k^2 + 136 = 0$, so $k^2 = 34$
$k = \pm\sqrt{34}$
Answer: $k = \pm\sqrt{34}$