Day 316 **S Tier 10-a-day: Model Solutions**

1 Sample space and conditional probability

A spinner numbered 1 to 4 and a spinner numbered 1 to 5 are each thrown once. The first score and the second score are recorded.

(a) Work out the probability that the two numbers are different.

(b) Given that at least one number is 3, work out the probability that the two numbers are different.

(a) There are 4 × 5 = 20 equally likely outcomes, and 16 of them satisfy: the two numbers are different

Probability = $\frac{4}{5}$

(b) 8 outcomes satisfy: at least one number is 3. Of these, 7 also satisfy: the two numbers are different

Probability = $\frac{7}{8}$

Answer: (a) $\frac{4}{5}$ (b) $\frac{7}{8}$

2 Angle in a semicircle

AB is a diameter of a circle, centre O. C is a point on the circle. Angle BAC = 67°.

(a) Work out the size of angle ACB.

(b) Work out the size of angle COB.

The diagram is not drawn accurately.

(a) The angle in a semicircle is 90°, so angle ACB = 90°

(b) OA = OC (radii), so triangle OAC is isosceles and angle OCA = 67°

Angle COB is an exterior angle of triangle OAC: 67 + 67 = 134°

(or: the angle at the centre is twice the angle at the circumference)

Answer: (a) 90° (b) 134°

3 Fractions

Work out $3\frac{1}{5} \times 1\frac{7}{8} - 4\frac{3}{5}$

Give your answer as a mixed number or a fraction in its simplest form.

Change the mixed numbers to improper fractions

$\frac{16}{5} \times \frac{15}{8} = 6$

$6 - \frac{23}{5} = \frac{7}{5}$

$= 1\frac{2}{5}$

Answer: $1\frac{2}{5}$

4 Probabilities with an unknown

A biased four-sided dice lands on one of four outcomes with these probabilities.

1: x; 2: 0.1; 3: x + 0.15; 4: 0.55

(a) Work out the value of x.

(b) The biased four-sided dice is used 200 times. Work out an estimate for the number of times it lands on 1.

(a) Probabilities add up to 1: x + 0.1 + x + 0.15 + 0.55 = 1

2x + 0.8 = 1, so 2x = 0.2

x = 0.1

(b) P(1) = x = 0.1, so 200 × 0.1 = 20

Answer: (a) x = 0.1 (b) 20

Day 316 **S Tier 10-a-day: Model Solutions**

5 Terms of a quadratic sequence

The nth term of a sequence is $n^2 + 6n + 3$.
(a) Work out the 12th term.
(b) Is 285 a term of the sequence? You must show your working.

(a) $12^2 + 6 \times 12 + 3 = 219$
(b) Solve $n^2 + 6n + 3 = 285$, so $n^2 + 6n - 282 = 0$
$b^2 - 4ac = 6^2 - 4 \times 1 \times (-282) = 1164$, which is not a square number
So n is not a whole number, and 285 is not a term
Answer: (a) 219 (b) No

6 Percentage change

A price is increased by 900%.
By what percentage must the new value be decreased to get back to the original value?

Let the original be 100. After a 900% increase it is 1000.
To return to 100, decrease by 900
Percentage decrease = $\frac{900}{1000} \times 100 = 90\%$
Answer: 90%

7 Shaded areas: exact answers

The diagram shows a square inside a circle of radius 7 cm.
The four vertices of the square lie on the circle.
Work out the area of the shaded region. Give your answer in terms of π.

The diagonal of the square is a diameter of the circle: 14 cm
Area of the square = $\frac{1}{2} \times$ diagonal $\times$ diagonal = $\frac{1}{2} \times 14 \times 14 = 98$
Area of circle = $\pi \times 7^2 = 49\pi$
Shaded area = $49\pi - 98$ cm^2
Answer: $(49\pi - 98)$ cm^2

8 Equations with algebraic fractions

Solve $\frac{3}{x+3} - \frac{6}{x+2} = \frac{1}{2}$

Multiply every term by $(x + 3)(x + 2)$: $3(x + 2) - 6(x + 3) = \frac{1}{2} \times (x + 3)(x + 2)$
Multiply by 2 to clear the fraction: $6(x + 2) - 12(x + 3) = (x + 3)(x + 2)$
Expand and rearrange so that one side is 0: $x^2 + 11x + 30 = 0$
Factorise: $(x + 5)(x + 6) = 0$
$x = -6$ or $x = -5$
Answer: $x = -6$ or $x = -5$

9 Proof with indices

Show that $2^{n+2} + 2^n$ is always a multiple of 5.

$2^{n+2} = 2^n \times 2^2$
$2^{n+2} + 2^n = 2^n(2^2 + 1)$
$2^2 + 1 = 5$
$= 5 \times 2^n$, which is a multiple of 5
Answer: 5×2^n

Day 316 **S Tier 10-a-day: Model Solutions**

10 Tangency conditions

The line $y = -2x + k$ is a tangent to the curve $y = 2x^2 - 3x - 2$.
Find the value of k.

Equate: $2x^2 - 3x - 2 = -2x + k$

Rearrange: $2x^2 - x - 2 - k = 0$

A tangent touches at one point, so $b^2 - 4ac = 0$: $1 - 4 \times 2 \times (-2 - k) = 0$

$8k + 17 = 0$

$k = -\frac{17}{8}$

Answer: $k = -\frac{17}{8}$