Day 314 **S Tier 10-a-day: Model Solutions**

1 Compound interest

Liam invests £7,500. The rates of interest for the next 3 years are 6%, 6% and 4%.
Compound interest is paid. Work out the value of the investment after 3 years.
Give your answer to the nearest penny. (Calculator)

Multipliers: 1.06, 1.06 and 1.04
7500 × 1.06 × 1.06 × 1.04 = 8764.08
To the nearest penny: £8,764.08
Answer: £8,764.08

2 Ordering numbers

Write these numbers in order of size, starting with the smallest.
$\frac{7}{8}$, $\frac{7}{15}$, 83%, 0.8

Convert each to a decimal: $\frac{7}{8}$ = 0.875, $\frac{7}{15}$ = 0.467, 83% = 0.83, 0.8 = 0.8

In order: $\frac{7}{15}$, 0.8, 83%, $\frac{7}{8}$

Answer: $\frac{7}{15}$, 0.8, 83%, $\frac{7}{8}$

3 Perimeter and area of a running track

The diagram shows a running track made from a rectangle and two semicircles.
(a) Work out the total length of the boundary of the track.
(b) Work out the area enclosed by the track. Give your answers in terms of π.
The diagram is not drawn accurately.

(a) The two semicircles make one full circle of radius 20: circumference = 2 × π × 20 = 40π
The two straight sides: 2 × 50 = 100
Boundary = (100 + 40π) m

(b) The rectangle is 50 m by 2 × 20 = 40 m, so its area is 50 × 40 = 2000. Circle: π × 20^2 = 400π
Area = (2000 + 400π) m^2

Answer: (a) (100 + 40π) m (b) (2000 + 400π) m^2

4 Where a line meets the axes

The line L has equation 7x + 24y = 168.
L crosses the x-axis at the point A and the y-axis at the point B.
(a) Find the coordinates of A and B.
(b) Work out the length of AB.

(a) At A, y = 0: 7x = 168, so x = 24. A is (24, 0)
At B, x = 0: 24y = 168, so y = 7. B is (0, 7)

(b) $AB^2 = 24^2 + 7^2 = 625$

AB = $\sqrt{625}$ = 25

Answer: (a) A(24, 0), B(0, 7) (b) 25

5 Exact trigonometry in triangles

ABC is a right-angled triangle.
Work out the exact value of x. Give your answer in its simplest form.
The diagram is not drawn accurately.

x is the adjacent side AB and the known side is the opposite side BC, so use tan 45°
tan 45° = 1
x = 5 ÷ tan 45° = 5 ÷ 1
x = 5
Answer: x = 5

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6 Using a line of best fit

The scatter graph shows hours of revision (x) and test mark (y).
The equation of the line of best fit is y = 0.5x + 3.
(a) Estimate y when x = 7.
(b) Explain why an estimate of y when x = 15 may be unreliable.

(a) $y = 0.5 \times 7 + 3 = 6.5$
(b) x = 15 is outside the range of the data on the graph, so the estimate relies on the pattern continuing (extrapolation)
Answer: (a) 6.5 (b) x = 15 is outside the range of the data

7 Rearranging when the subject appears twice

Make u the subject of $v = \frac{6u + 3}{2u + 6}$

Multiply by 2u + 6: $v(2u + 6) = 6u + 3$
Expand: $2uv + 6v = 6u + 3$
Collect the u terms: $2uv - 6u = 3 - 6v$
Factorise: $u(2v - 6) = -6v + 3$
$u = \frac{-6v + 3}{2v - 6}$
Answer: $u = \frac{-6v + 3}{2v - 6}$

8 Equations with algebraic fractions

Solve $\frac{5}{x + 2} - \frac{2}{x + 5} = \frac{3}{4}$

Multiply every term by (x + 2)(x + 5): $5(x + 5) - 2(x + 2) = \frac{3}{4} \times (x + 2)(x + 5)$
Multiply by 4 to clear the fraction: $20(x + 5) - 8(x + 2) = 3(x + 2)(x + 5)$
Expand and rearrange so that one side is 0: $x^2 + 3x - 18 = 0$
Factorise: $(x - 3)(x + 6) = 0$
$x = -6$ or $x = 3$
Answer: $x = -6$ or $x = 3$

9 Bounds and accuracy

a = 3.84 and b = 82, both correct to 3 significant figures.
Work out a ÷ b.
Write down your answer to a suitable degree of accuracy.
You must show your working and give a reason for your answer. (Calculator)

$3.835 \leq a < 3.845$; $81.95 \leq b < 82.05$
Lower bound = 3.835 ÷ 82.05 = 0.046740...
Upper bound = 3.845 ÷ 81.95 = 0.046919...
Both bounds round to 0.047 to 2 significant figures, but not to 3 significant figures
Answer: 0.047 (2 s.f.)

10 Loci and regions

ABCD is a rectangle with AB = 16 cm and AD = 18 cm.
A point P inside the rectangle is such that AP ≤ 12 cm and P is nearer to AB than to AD.
Work out the area of the region in which P can lie. Give your answer in terms of π.

AP ≤ 12 cm: P is inside the quarter circle of radius 12 cm centred at A (it fits inside the rectangle)
Nearer to AB than AD: P is on the AB side of the angle bisector of angle DAB, which is the line through A at 45° to AB
The bisector cuts the quarter circle into two equal halves
Area $= \frac{1}{2} \times \frac{1}{4} \times \pi \times 12^2 = 18\pi$ cm^2
Answer: 18π cm^2