Day 313 **S Tier 10-a-day: Model Solutions**

1 LCM problems

A red light flashes every 4 minutes.
A blue light flashes every 6 minutes.
A green light flashes every 7 minutes.
They all flash together at 10:00 am.
(a) At what time do they next flash together?

Find the LCM of 4, 6 and 7
$4 = 2^2$, $6 = 2 \times 3$, $7 = 7$
LCM $= 2^2 \times 3 \times 7 = 84$
84 minutes after 10:00 am is 11:24 am
Answer: 11:24 am

2 Factorising quadratics

Factorise $6x^2 + 23x + 21$

$6 \times 21 = 126$. Two numbers that multiply to 126 and add to 23 are 9 and 14
Split the middle term: $6x^2 + 9x + 14x + 21$
Factorise in pairs: $3x(2x + 3) + 7(2x + 3)$
$= (3x + 7)(2x + 3)$
Answer: $(3x + 7)(2x + 3)$

3 Rational and irrational numbers

Here are six numbers.

$\sqrt{0.25}$, $3 + \sqrt{12}$, $\sqrt[3]{27}$, $\sqrt{2} + \sqrt{8}$, $\pi \div 2$, $\frac{\sqrt{18}}{\sqrt{2}}$

Write down the irrational numbers.

Simplify each number:
$\sqrt{0.25} = 0.5$; $3 + \sqrt{12} = 3 + 2\sqrt{3}$; $\sqrt[3]{27} = 3$
$\sqrt{2} + \sqrt{8} = 3\sqrt{2}$; $\pi \div 2$ is irrational; $\frac{\sqrt{18}}{\sqrt{2}} = \sqrt{9} = 3$
Answer: $3 + \sqrt{12}$, $\sqrt{2} + \sqrt{8}$, $\pi \div 2$

4 Relative speed

Isla and Tariq live 260 km apart.
At 10:00 they set off towards each other on the same road.
Isla drives at 45 km/h and Tariq drives at 85 km/h.
(a) At what time do they meet?
(b) How far from Isla's home do they meet?

Closing speed = 45 + 85 = 130 km/h
(a) Time = 260 ÷ 130 = 2 hours = 120 minutes, so they meet at 12:00
(b) Distance = 45 × 2 = 90 km
Answer: (a) 12:00 (b) 90 km

5 Speed-time graphs

The graph shows the speed of a train between two stations.
(a) Work out the acceleration in the first 4 seconds.
(b) Work out the total distance.

(a) Acceleration = change in speed ÷ time = 12 ÷ 4 = 3 m/s^2
(b) Distance = area under the graph, split into two triangles and a rectangle
$\frac{1}{2} \times 4 \times 12 + 12 \times 12 + \frac{1}{2} \times 4 \times 12 = 24 + 144 + 24$
$= 192$
Answer: (a) 3 m/s^2 (b) 192 m

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6 Ratio and algebra

In a coach, the ratio of the number of adults to the number of children is 7 : 5.
Then 5 more adults are added and 17 children are removed.
The ratio is now 2 : 1.
How many adults were there at the start?

Let the numbers at the start be 7k and 5k
(7k + 5) : (5k − 17) = 2 : 1, so 1(7k + 5) = 2(5k − 17)
7k + 5 = 10k − 34
−3k = −39, so k = 13
Number of adults at the start: 7 × 13 = 91
Answer: 91

7 Perpendicular bisector

A is the point (0, −4) and B is the point (2, −12).
Find an equation of the perpendicular bisector of AB. Give your answer in the form ax + by = c, where a, b and c are integers.

Midpoint of AB = (1, −8)
Gradient of AB = −4
Perpendicular gradient = $\frac{1}{4}$
$y - (-8) = \frac{1}{4}(x - 1)$
Rearrange to integer coefficients: x − 4y = 33
Answer: x − 4y = 33

8 Histograms with an unmarked scale

The histogram shows the times of people spent waiting at a clinic.
120 of them had a time between 40 and 70 minutes.
(a) How many had a time between 20 and 40 minutes?
(b) How many people are there in total?

Frequency is proportional to the area of a bar
40 to 70: bar is 2 squares high and 30 wide, area 60, which represents 120
So 1 unit of area represents 120 ÷ 60 = 2
(a) 20 to 40: 5 × 20 = 100 units of area, so 100 × 2 = 200
(b) Total area = 10 + 40 + 100 + 60 = 210, so total = 210 × 2 = 420
Answer: (a) 200 (b) 420

9 HCF and LCM

The HCF of two numbers is 5 and their LCM is 225.
One of the numbers is 45.
Work out the other number.

HCF × LCM = product of the two numbers
5 × 225 = 1125
Other number = 1125 ÷ 45 = 25
Answer: 25

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10 Loci and regions

ABCD is a rectangle with AB = 4 cm and AD = 7 cm.
A point P inside the rectangle is such that AP ≤ 4 cm and P is nearer to A than to B.
Work out the area of the region in which P can lie. Give your answer in terms of π and surds.

Nearer to A than to B: P is on the A side of the perpendicular bisector of AB, a distance 2 cm from A
AP ≤ 4 cm: P is inside the quarter circle of radius 4 cm centred at A

The bisector meets the arc where $\cos\theta = \frac{2}{4}$, so $\theta = 60°$ at A. The region is a right-angled triangle plus a sector of angle 30°

Triangle: base 2 cm, height $4\sin 60° = 2\sqrt{3}$ cm, area $= \frac{1}{2} \times 2 \times 2\sqrt{3} = 2\sqrt{3}$

Sector of angle 30° : $\frac{30}{360} \times \pi \times 4^2 = \frac{4}{3}\pi$

Area $= 2\sqrt{3} + \frac{4}{3}\pi$ cm^2

Answer: $(2\sqrt{3} + \frac{4}{3}\pi)$ cm^2