Day 312 **S Tier 10-a-day: Model Solutions**

1 Stratified sampling

The table shows the number of members in each of four groups.
(a) A stratified sample of 33 is taken. Work out how many come from Group B.
(b) A different stratified sample has 33 from Group A. Work out the size of this sample.

Total = 220 + 140 + 80 + 220 = 660

(a) Group B: $\frac{140}{660} \times 33 = 7$

(b) Fraction sampled = 33 ÷ 220 = $\frac{3}{20}$, so sample size = $\frac{3}{20} \times 660 = 99$

Answer: (a) 7 (b) 99

2 Time zones

A flight leaves London at 08:15 (London time).
It takes 11 hours 45 minutes.
When it is 12:00 in London it is 20:00 in Tokyo.
At what time, in Tokyo time, does the flight land? Say if it is on a different day.

Arrival in London time = 08:15 + 11 h 45 min = 20:00
Tokyo is 8 hours ahead of London, so add 8 hours
04:00 the next day
Answer: 04:00 the next day

3 Expanding three brackets

Expand and simplify $(x + 2)(x + 6)(x - 2)$

Expand the first two brackets: $(x + 2)(x + 6) = x^2 + 8x + 12$
Multiply by $(x - 2)$: $x(x^2 + 8x + 12) - 2(x^2 + 8x + 12)$
$= x^3 + 6x^2 - 4x - 24$
Answer: $x^3 + 6x^2 - 4x - 24$

4 Bounds of area and perimeter

A rectangle has length 30 cm and width 11 cm.
Both measurements are correct to the nearest centimetre.
Kofi says that the area of the rectangle could be exactly 351 cm^2.
Is Kofi correct? You must show your working.

29.5 ≤ length < 30.5 and 10.5 ≤ width < 11.5
Lower bound of area = 29.5 × 10.5 = 309.75
Upper bound of area = 30.5 × 11.5 = 350.75
So 309.75 ≤ area < 350.75
351 is outside this interval, so Kofi is not correct
Answer: No, 351 is outside the interval

5 Speed-time graphs

The graph shows the speed of a car during 20 seconds.
(a) Work out the acceleration during the first 4 seconds.
(b) Work out the average speed over the whole 20 seconds.

(a) Acceleration = change in speed ÷ time = (20 − 8) ÷ 4 = 3 m/s^2
(b) Distance = area under the graph
Trapezium: $\frac{8 + 20}{2} \times 4 = 56$; rectangle: 20 × 9 = 180; triangle: $\frac{1}{2} \times 7 \times 20 = 70$
Total = 306 m, so average speed = 306 ÷ 20 = 15.3 m/s
Answer: (a) 3 m/s^2 (b) 15.3 m/s

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6 Exponential graphs

The diagram shows the graph of $y = ka^x$, where k and a are constants and $a > 0$.
The curve passes through the points (1, 21) and (3, 189).
Find the value of k and the value of a.

Substitute the points: $k\,a^1 = 21$ and $k\,a^3 = 189$

Divide: $a^2 = \frac{189}{21} = 9$

$a = 3$ (positive, since $a > 0$)
From the first point: $k \times 3 = 21$, so $k = 7$
Answer: k = 7, a = 3

7 Pythagoras with algebra

ABC is a right-angled triangle with sides $(2x - 3)$ cm, $(x - 1)$ cm and $(x + 8)$ cm. The longest side is $(x + 8)$ cm.
(a) Find the value of x.
(b) Work out the area of the triangle.
The diagram is not drawn accurately.

(a) Pythagoras: $(2x - 3)^2 + (x - 1)^2 = (x + 8)^2$
Expand and collect terms: $4x^2 - 30x - 54 = 0$
Factorise: $2(x - 9)(2x + 3) = 0$

$x = 9$ or $x = -\frac{3}{2}$, and x must be positive, so $x = 9$

(b) The two shorter sides are 15 cm and 8 cm

Area $= \frac{1}{2} \times 15 \times 8 = 60$ cm^2

Answer: (a) x = 9 (b) 60 cm^2

8 Reverse percentages

The value of a house is increased by 20% in one year.
The next year its value is decreased by 5%.
The next year its value is increased by 8%.
It is now worth £147,744.
Work out the value of the house before the first change.

Multipliers: 1.2, 0.95, 1.08
Overall multiplier = $1.2 \times 0.95 \times 1.08 = 1.2312$
Original = final ÷ multiplier = £147,744 ÷ 1.2312
= £120,000
Answer: £120,000

9 Loci and regions

ABCD is a rectangle with AB = 5 cm and AD = 8 cm.
A point P inside the rectangle is such that AP ≤ 2 cm and P is nearer to AB than to AD.
Work out the area of the region in which P can lie. Give your answer in terms of π.

AP ≤ 2 cm: P is inside the quarter circle of radius 2 cm centred at A (it fits inside the rectangle)
Nearer to AB than AD: P is on the AB side of the angle bisector of angle DAB, which is the line through A at 45° to AB
The bisector cuts the quarter circle into two equal halves

Area $= \frac{1}{2} \times \frac{1}{4} \times \pi \times 2^2 = \frac{1}{2}\pi$ cm^2

Answer: $\frac{1}{2}\pi$ cm^2

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10 Transformations of graphs

The point P(8, 5) lies on the graph of y = f(x).
Write down the coordinates of the image of P on the graph of
(a) y = f(−x) (b) y = f(x) − 5

(a) y = f(−x) is a reflection in the y-axis, so the x coordinate changes sign: (−8, 5)
(b) y = f(x) − 5 is a translation of 5 down: (8, 0)
Answer: (a) (−8, 5) (b) (8, 0)