Day 311 **S Tier 10-a-day: Model Solutions**

1 Substitution

Work out the value of $3p - q^3$
when $p = -4$, $q = 2$ and $r = -1$.

Substitute the values: $3 \times (-4) - 2^3$
$= -12 - 8$
$= -20$
Answer: -20

2 Chords and Pythagoras in circles

AB is a chord of a circle, centre O. M is the midpoint of AB. (Calculator)
(a) Work out the length of the chord AB.
(b) Work out angle AOB, to 1 decimal place.
The diagram is not drawn accurately.

(a) $AM^2 = OA^2 - OM^2 = 29^2 - 21^2 = 400$
AM = 20 cm
AB = 2 × 20 = 40 cm
(b) sin(angle AOM) = $\frac{20}{29}$
Angle AOM = 43.60°, so angle AOB = 2 × 43.60 = 87.2°
Answer: (a) 40 cm (b) 87.2°

3 Tests of divisibility

The 4-digit number 1k58 is a multiple of 6.
k is a single digit.
Find all the possible values of k.

A multiple of 6 is even and a multiple of 3
The last digit is 8, which is even
Digit sum = 1 + 5 + 8 + k = 14 + k
This must be a multiple of 3, so k = 1 or 4 or 7
Answer: k = 1 or 4 or 7

4 Pressure

A wooden block in the shape of a cuboid weighs 990 N.
Its smallest face has area 100 cm^2. Its largest face has area 3,000 cm^2.
Pressure = force ÷ area
Work out (a) the greatest pressure and (b) the least pressure the wooden block can exert on the ground, in N/m^2.

1 m^2 = 10 000 cm^2, so 100 cm^2 = 0.01 m^2 and 3,000 cm^2 = 0.3 m^2
(a) Greatest pressure is on the smallest face: 990 ÷ 0.01 = 99,000 N/m^2
(b) Least pressure is on the largest face: 990 ÷ 0.3 = 3,300 N/m^2
Answer: (a) 99,000 N/m^2 (b) 3,300 N/m^2

5 Cosine rule and area of a triangle

The diagram shows triangle ABC. (Calculator)
(a) Work out angle ACB, marked θ, to 1 decimal place.
(b) Work out the area of triangle ABC to 3 significant figures.
The diagram is not drawn accurately.

(a) $\cos\theta = \frac{6^2 + 4^2 - 6^2}{2 \times 6 \times 4} = \frac{16}{48}$
θ = 70.5° (1 d.p.)
(b) Area = $\frac{1}{2} \times 6 \times 4 \times \sin 70.5°$
= 11.3 cm^2 (3 s.f.)
Answer: (a) 70.5° (b) 11.3 cm^2

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6 Perpendicular lines

The diagram shows the straight line L, which passes through the points A and B.
The line M is perpendicular to L and passes through B.
Find the equation of M in the form y = mx + c.

Gradient of L = $\frac{6-0}{0-(-3)} = 2$

Perpendicular gradient = $-1 \div 2 = -\frac{1}{2}$

M passes through B(−3, 0): $0 = (-\frac{1}{2}) \times (-3) + c$, so $c = -\frac{3}{2}$

$y = -\frac{1}{2}x - \frac{3}{2}$

Answer: $y = -\frac{1}{2}x - \frac{3}{2}$

7 Independent events and probability rules

A and B are two events.
P(A) = 0.3, P(B) = 0.4 and P(A or B) = 0.5
(a) Work out P(A and B).
(b) Are A and B independent? You must show your working.

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$0.5 = 0.3 + 0.4 - P(A \cap B)$, so $P(A \cap B) = 0.2$
(b) $P(A) \times P(B) = 0.3 \times 0.4 = 0.12$
This is not equal to 0.2, so A and B are not independent
Answer: (a) 0.2 (b) No

8 Line of best fit

The scatter graph shows temperature (x) and ice creams sold (y).
The line of best fit passes through (2, 8) and (6, 20).
(a) Find its equation in the form y = mx + c.
(b) Explain what its gradient tells you.

(a) Gradient $m = \frac{20-8}{6-2} = \frac{12}{4} = 3$
y = 3x + c: using (2, 8), $8 = 3 \times 2 + c$, so c = 2
y = 3x + 2
(b) For each extra degree of temperature, the number of ice creams sold goes up by 3
Answer: (a) y = 3x + 2 (b) For each extra degree of temperature, the number of ice creams sold goes up by 3

9 Percentages with an unknown rate

The price of a car is decreased by x%.
The new price is then increased by x%.
The final price is 1% less than the original price.
Work out the value of x.

Let the original price be 1. Multipliers: $(1 + \frac{x}{100})$ and $(1 - \frac{x}{100})$

$(1 + \frac{x}{100})(1 - \frac{x}{100}) = 1 - \frac{x^2}{10000}$

Final price is 1% less, so the multiplier is 0.99: $1 - \frac{x^2}{10000} = 0.99$

$\frac{x^2}{10000} = 0.01$, so $x^2 = 100$ and x = 10
Answer: x = 10

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10 Area of a triangle using sine

Triangle ABC has AB = 20 cm and BC = 4 cm.

The area of triangle ABC is 20 cm^2.

Angle ABC is obtuse.

Work out the size of angle ABC.

Do not use a calculator.

Area = $\frac{1}{2} \times AB \times BC \times \sin B$

$20 = \frac{1}{2} \times 20 \times 4 \times \sin B$

$\sin B = 20 \div 40 = \frac{1}{2}$

$\sin 30° = \frac{1}{2}$, and B is obtuse, so $B = 180° - 30° = 150°$

Answer: 150°