Day 310 **S Tier 10-a-day: Model Solutions**

1 Laws of indices

Given that $\frac{5^3 \times 25}{25^2} = 5^n$,

find the value of n.

$25 = 5^2$

Numerator: $5^3 \times 5^2 = 5^5$

Denominator: 5^4

Subtract the powers: $5 - 4 = 1$

Answer: n = 1

2 Comparing costs

Sparkle Cleaners has a call-out fee of £5 plus £2 per hour.
Fresh Start has a call-out fee of £12.50 plus £1.50 per hour.
(a) Work out the number of hours for which the two costs are the same.
(b) Which is cheaper for 24 hours, and by how much?

(a) Let n be the number of hours. Working in pounds, the costs are equal when $5 + 2n = 12.50 + 1.50n$
$0.50n = 7.50$, so $n = 15$

(b) Sparkle Cleaners: £5 + £2 × 24 = £53

Fresh Start: £12.50 + £1.50 × 24 = £48.50

Fresh Start is cheaper by £4.50

Answer: (a) 15 (b) Fresh Start, by £4.50

3 Hollow cylinder and density

A metal pipe is a hollow cylinder of length 25 cm. (Calculator)
The outer radius is 6 cm and the inner radius is 5 cm.
The density of the metal is 8.9 g/cm^3.
Work out the mass of the pipe. Give your answer in kilograms correct to 3 significant figures.

Volume of metal = $\pi \times 6^2 \times 25 - \pi \times 5^2 \times 25 = \pi(36 - 25) \times 25$

$= 863.94$ cm^3

Mass = density × volume = 8.9 × 863.94 = 7689.0 g

= 7.69 kg (3 s.f.)

Answer: 7.69 kg

4 Probabilities with an unknown

A biased spinner lands on one of four outcomes with these probabilities.
red: 3x; blue: 2x; green: x + 0.15; yellow: 0.25
(a) Work out the value of x.
(b) The biased spinner is used 100 times. Work out an estimate for the number of times it lands on red.

(a) Probabilities add up to 1: $3x + 2x + x + 0.15 + 0.25 = 1$

$6x + 0.4 = 1$, so $6x = 0.6$

$x = 0.1$

(b) P(red) = 3 × 0.1 = 0.3, so 100 × 0.3 = 30

Answer: (a) x = 0.1 (b) 30

5 Standard form problems

The mass of one grain of sand is 8×10^{-2} g.
A bucket of sand has a total mass of 5.12×10^5 g.
Work out how many there are in the bucket.
Give your answer in standard form.

Number = total mass ÷ mass of one

$(5.12 \times 10^5) \div (8 \times 10^{-2})$

$5.12 \div 8 = 0.64$ and $10^5 \div 10^{-2} = 10^7$

$= 6.4 \times 10^6$

Answer: 6.4×10^6

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6 Geometric sequences

The 3rd term of a geometric sequence is 9.
The 5th term of the sequence is 1.
All the terms of the sequence are positive.
(a) Work out the common ratio.
(b) Work out the first term.

(a) The common ratio r satisfies $r^2 = 1 \div 9 = \frac{1}{9}$

$r = \sqrt{\frac{1}{9}} = \frac{1}{3}$ (positive, since all terms are positive)

(b) The 3rd term is $a \times r^2$: $a \times \frac{1}{9} = 9$

$a = 9 \div \frac{1}{9} = 81$

Answer: (a) $r = \frac{1}{3}$ (b) 81

7 Inverse proportion

T is inversely proportional to the cube of n.
When n = 4, T = 1.
Find the value of T when n = 1.

$T = \frac{k}{n^3}$

$1 = \frac{k}{64}$, so $k = 1 \times 64 = 64$

$T = \frac{64}{1} = 64$

Answer: T = 64

8 Vector proof: collinear points

OA = **a** and OB = **b**.
OP = 2**a** − 3**b**, OQ = 8**a** + 6**b** and OR = 16**a** + 18**b**.
(a) Show that P, Q and R lie on a straight line.
(b) Work out the ratio PQ : QR in its simplest form.

(a) PQ = OQ − OP = 6**a** + 9**b**
QR = OR − OQ = 8**a** + 12**b**

QR = $\frac{4}{3}$ PQ, so PQ and QR are parallel, and they share the point Q

So P, Q and R lie on a straight line
(b) PQ : QR = 3 : 4
Answer: (b) 3 : 4

9 Estimating the mean from a histogram table

The table gives information about the times of some people.
Work out an estimate for the mean time.
Give your answer correct to 1 decimal place. (Calculator)

Frequency = frequency density × class width: 18, 7, 38, 20
Mid-points: 5, 15, 30, 50
Σfm = 18×5 + 7×15 + 38×30 + 20×50 = 2335, Σf = 83
Mean = 2335 ÷ 83 = 28.1
Answer: 28.1

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10 Areas of similar triangles

In the diagram, DE is parallel to BC. AD = 8 cm, DB = 4 cm and the area of triangle ADE is 80 cm^2.
Work out the area of the trapezium DBCE.
The diagram is not drawn accurately.

Triangles ADE and ABC are similar. AB = 8 + 4 = 12 cm

Length scale factor = $12 \div 8 = \frac{3}{2}$

Area scale factor = $(\frac{3}{2})^2 = \frac{9}{4}$

Area of triangle ABC = $80 \times \frac{9}{4} = 180$ cm^2

Trapezium = $180 - 80 = 100$ cm^2

Answer: 100 cm^2