Day 31 **S Tier 10-a-day: Model Solutions**

1 Inverse functions

$f(x) = \frac{8}{x-4} + 2$

(a) Find $f^{-1}(x)$.

(b) Find the value of $f^{-1}(-6)$.

(a) Let $y = \frac{8}{x-4} + 2$, so $y - 2 = \frac{8}{x-4}$

Make x the subject: $x - 4 = \frac{8}{y-2}$

$x = \frac{8}{y-2} + 4$

$f^{-1}(x) = \frac{8}{x-2} + 4$

(b) $f^{-1}(-6) = \frac{8}{-6-2} + 4 = 3$

Answer: (a) $f^{-1}(x) = \frac{8}{x-2} + 4$ (b) 3

2 Mean of a changing set

The mean of 6 numbers is 129.
One number is removed and the mean of the remaining 5 numbers is 146.8.
Work out the number that was removed.

Total of the 6 numbers = 6 × 129 = 774
Total of the 5 remaining = 5 × 146.8 = 734
Number removed = 774 − 734 = 40
Answer: 40

3 Fractional and negative indices

Work out $(\frac{64}{125})^{\frac{7}{3}}$

The denominator 3 means the cube root: the cube root of $\frac{64}{125}$ is $\frac{4}{5}$

The numerator 7 means raise to the power 7: $(\frac{4}{5})^7 = \frac{16384}{78125}$

Answer: $\frac{16384}{78125}$

4 Finding the rate of interest

Priya invested £9,500 in a savings account for 2 years.
Compound interest was paid at a fixed rate per year.
At the end of the 2 years the account held £10,674.20.
Work out the annual rate of interest. (Calculator)

Let the yearly multiplier be m, so $9500 \times m^2 = 10674.20$
$m^2 = 10674.20 \div 9500 = 1.1236$
$m = \sqrt{1.1236} = 1.06$
Rate = (1.06 − 1) × 100 = 6%
Answer: 6% per year

5 Tangents and circle theorems

PA and PB are tangents to a circle, centre O. C is a point on the circle. Angle ACB = 54°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle APB.
The diagram is not drawn accurately.

(a) The angle at the centre is twice the angle at the circumference: 2 × 54 = 108°
(b) Angles OAP and OBP are 90° (a tangent meets a radius at 90°)
Angle APB = 360 − 90 − 90 − 108 = 72°
Answer: (a) 108° (b) 72°

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6 Compound interest

Ivan invests £6,000.
The account pays compound interest of 4% a year for the first 3 years, then 2.5% a year for the next 2 years.
Work out the total amount after 5 years, to the nearest penny. (Calculator)

Multipliers: 1.04 for 3 years, then 1.025 for 2 years
£6,000 × 1.04^3 × 1.025^2
= £7,090.86
Answer: £7,090.86

7 Forming and solving quadratic equations

The sum of the squares of two consecutive positive integers is 85.
Let the smaller integer be n.
(a) Show that $n^2 + n - 42 = 0$
(b) Work out the two integers.

(a) $n^2 + (n + 1)^2 = 85$, so $2n^2 + 2n + 1 = 85$
$2n^2 + 2n = 84$, so $n^2 + n - 42 = 0$
(b) Factorise: $(n - 6)(n + 7) = 0$
$n = 6$ or $n = -7$. n is positive, so $n = 6$
The integers are 6 and 7
Answer: (b) 6 and 7

8 Vectors in a triangle

OAB is a triangle. OA = **a** and OB = **b**.
M is on OA with OM : MA = 2 : 1. N is on AB with AN : NB = 1 : 2.
(a) Find ON in terms of **a** and **b**.
(b) Show that MN is parallel to OB.
The diagram is not drawn accurately.

(a) AB = $-\mathbf{a} + \mathbf{b}$, so AN = $\frac{1}{3}(\mathbf{b} - \mathbf{a})$

ON = OA + AN = $\mathbf{a} + \frac{1}{3}(\mathbf{b} - \mathbf{a}) = \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$

(b) OM = $\frac{2}{3}\mathbf{a}$

MN = ON − OM = $\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b} - \frac{2}{3}\mathbf{a} = \frac{1}{3}\mathbf{b}$

MN = $\frac{1}{3}\mathbf{b}$, which is a multiple of OB, so MN is parallel to OB

Answer: (a) $\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$ (b) MN = $\frac{1}{3}\mathbf{b}$, parallel to OB

9 HCF and LCM

The HCF of two numbers is 3 and their LCM is 72.
The smaller number is 9.
Work out the larger number.

HCF × LCM = product of the two numbers
3 × 72 = 216
Larger number = 216 ÷ 9 = 24
Answer: 24

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10 Loci and regions

ABCD is a rectangle with AB = 8 cm and AD = 13 cm.
A point P inside the rectangle is such that AP ≤ 8 cm and P is nearer to A than to B.
Work out the area of the region in which P can lie. Give your answer in terms of π and surds.

Nearer to A than to B: P is on the A side of the perpendicular bisector of AB, a distance 4 cm from A
AP ≤ 8 cm: P is inside the quarter circle of radius 8 cm centred at A

The bisector meets the arc where $\cos\theta = \frac{4}{8}$, so $\theta = 60°$ at A. The region is a right-angled triangle plus a sector of angle 30°

Triangle: base 4 cm, height $8\sin 60° = 4\sqrt{3}$ cm, area $= \frac{1}{2} \times 4 \times 4\sqrt{3} = 8\sqrt{3}$

Sector of angle 30° : $\frac{30}{360} \times \pi \times 8^2 = \frac{16}{3}\pi$

Area $= 8\sqrt{3} + \frac{16}{3}\pi$ cm^2

Answer: $\left(8\sqrt{3} + \frac{16}{3}\pi\right)$ cm^2