Day 309 **S Tier 10-a-day: Model Solutions**

1 Equations with fractions

Solve $\frac{x-2}{6} - \frac{x-6}{2} = 2$

Multiply every term by 6: $(x - 2) - 3(x - 6) = 12$
Expand: $-2x + 16 = 12$
$-2x = -4$
$x = 2$
Answer: x = 2

2 Angles in polygons

The interior angles of a hexagon are
$x°$, $(2x - 25)°$, $(2x - 5)°$, $(2x - 30)°$, $(x + 10)°$, $(x + 5)°$
(a) Work out the value of x.
(b) Work out the size of the largest angle.

(a) Sum of interior angles = $(6 - 2) \times 180 = 720°$
$9x - 45 = 720$, so $9x = 765$
$x = 85$
(b) The angles are 85°, 145°, 165°, 140°, 95°, 90°
The largest is 165°
Answer: (a) x = 85 (b) 165°

3 Fractions

Work out $1\frac{2}{3} \div 2\frac{1}{2} + 1\frac{4}{5}$

Give your answer as a mixed number or a fraction in its simplest form.

Change the mixed numbers to improper fractions
$\frac{5}{3} \div \frac{5}{2} = \frac{5}{3} \times \frac{2}{5} = \frac{2}{3}$
$\frac{2}{3} + \frac{9}{5} = \frac{37}{15}$
$= 2\frac{7}{15}$
Answer: $2\frac{7}{15}$

4 Pressure

A metal crate in the shape of a cuboid weighs 360 N.
Its smallest face has area 1000 cm^2. Its largest face has area 1,500 cm^2.
Pressure = force ÷ area
Work out (a) the greatest pressure and (b) the least pressure the metal crate can exert on the ground, in N/m^2.

1 m^2 = 10 000 cm^2, so 1000 cm^2 = 0.1 m^2 and 1,500 cm^2 = 0.15 m^2
(a) Greatest pressure is on the smallest face: $360 \div 0.1 = 3{,}600$ N/m^2
(b) Least pressure is on the largest face: $360 \div 0.15 = 2{,}400$ N/m^2
Answer: (a) 3,600 N/m^2 (b) 2,400 N/m^2

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5 Column vectors

$\mathbf{a} = \begin{pmatrix} 0 \\ -2 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$\mathbf{a} + k\mathbf{b}$ is parallel to $\mathbf{c}$.

Work out the value of k.

$\mathbf{a} + k\mathbf{b} = \begin{pmatrix} 0 + 2k \\ -2 + 4k \end{pmatrix}$

Parallel to $\mathbf{c}$: $\frac{0 + 2k}{2} = \frac{-2 + 4k}{3}$

$3(0 + 2k) = 2(-2 + 4k)$

$k \times (-2) = -4$

$k = 2$

Answer: k = 2

6 HCF and LCM

(a) Write 50 and 22 as products of prime factors.

(b) Find the HCF and the LCM of 50 and 22.

Give your answers as numbers.

(a) $50 = 2 \times 5^2$ and $22 = 2 \times 11$

(b) HCF $= 2 = 2$

LCM $= 2 \times 5^2 \times 11 = 550$

Answer: (a) 2×5^2; 2×11 (b) HCF = 2, LCM = 550

7 Geometric sequences

The 2nd term of a geometric sequence is 15.

The 5th term of the sequence is 405.

(a) Work out the common ratio.

(b) Work out the first term.

(a) The common ratio r satisfies $r^3 = 405 \div 15 = 27$

$r = \sqrt[3]{27} = 3$

(b) The 2nd term is $a \times r^1$: $a \times 3 = 15$

$a = 15 \div 3 = 5$

Answer: (a) r = 3 (b) 5

8 Rationalising the denominator

Write $\frac{5 + \sqrt{2}}{2 + \sqrt{2}}$ in the form $p + q\sqrt{2}$

where p and q are rational numbers.

Multiply top and bottom by $(2 - \sqrt{2})$:

Numerator: $(5 + \sqrt{2})(2 - \sqrt{2}) = 8 - 3\sqrt{2}$

Denominator: $4 - 2 = 2$

$= \frac{8 - 3\sqrt{2}}{2}$

Answer: $\frac{8 - 3\sqrt{2}}{2}$

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9 Pythagoras with algebra

ABC is a right-angled triangle with sides (2x + 2) cm, (x) cm and (x + 8) cm. The longest side is (x + 8) cm.
(a) Find the value of x.
(b) Work out the area of the triangle.
The diagram is not drawn accurately.

(a) Pythagoras: $(2x + 2)^2 + x^2 = (x + 8)^2$
Expand and collect terms: $4x^2 - 8x - 60 = 0$
Factorise: $4(x - 5)(x + 3) = 0$
$x = 5$ or $x = -3$, and x must be positive, so $x = 5$
(b) The two shorter sides are 12 cm and 5 cm
Area = $\frac{1}{2} \times 12 \times 5 = 30$ cm^2

Answer: (a) x = 5 (b) 30 cm^2

10 Venn diagrams with algebra

The Venn diagram shows how many of 15 students have a tablet (T) or a laptop (L).
(a) Work out the value of x.
(b) A student is chosen at random from those who have a laptop. Work out the probability that this student also has a tablet.

(a) Total: $(x + 2) + x + (2x - 1) + 2 = 15$
$4x + 3 = 15$, so $x = 3$
(b) Students in the second circle: $x + 2x - 1 = 3 + 5 = 8$, of whom 3 are in both
Probability = $\frac{3}{8}$

Answer: (a) x = 3 (b) $\frac{3}{8}$