Day 308 **S Tier 10-a-day: Model Solutions**

1 Area and quadratic equations

The diagram shows a rectangle of length $(3x + 5)$ cm, width $(x - 2)$ cm and area 174 cm^2.
(a) Show that $3x^2 - x - 184 = 0$.
(b) Work out the perimeter of the rectangle.

(a) $(3x + 5)(x - 2) = 174$
Expand: $3x^2 - x - 10 = 174$, so $3x^2 - x - 184 = 0$
(b) Factorise: $(x - 8)(3x + 23) = 0$
$x = 8$ ($x = -\frac{23}{3}$ is not possible as a length would be negative)
Length = 29 cm and width = 6 cm
Perimeter = $2 \times (29 + 6) = 70$ cm
Answer: (b) 70 cm

2 Stratified sampling

The table shows the number of members in each of four groups.
(a) A stratified sample of 31 is taken. Work out how many come from Group C.
(b) A different stratified sample has 27 from Group C. Work out the size of this sample.

Total = 160 + 60 + 180 + 220 = 620
(a) Group C: $\frac{180}{620} \times 31 = 9$
(b) Fraction sampled = $27 \div 180 = \frac{3}{20}$, so sample size = $\frac{3}{20} \times 620 = 93$
Answer: (a) 9 (b) 93

3 Simplifying surds

Write $\sqrt{24} + \sqrt{96} + \sqrt{6}$ in the form $a\sqrt{6}$
where a is an integer.

$\sqrt{24} = 2\sqrt{6}$; $\sqrt{96} = 4\sqrt{6}$
$(2 + 4 + 1)\sqrt{6} = 7\sqrt{6}$
Answer: $7\sqrt{6}$

4 Completing the square

(a) Write $-9 - 8x - x^2$ in the form $a - (x + b)^2$
(b) Write down the maximum value of $-9 - 8x - x^2$ and the value of x for which it occurs.

(a) $-9 - 8x - x^2 = -(x^2 + 8x) - 9$
$x^2 + 8x = (x + 4)^2 - 16$
so $-9 - 8x - x^2 = -(x + 4)^2 + 16 - 9 = 7 - (x + 4)^2$
(b) The square is never negative, so the maximum is 7 when $x = -4$
Answer: (a) $7 - (x + 4)^2$ (b) maximum 7 at $x = -4$

5 Similar shapes

Two tins are mathematically similar.
The heights of the tins are 4 cm and 10 cm.
The smaller one has volume 80 cm^3.
Work out the volume of the larger one.

Scale factor of lengths = $10 \div 4 = \frac{5}{2}$
Scale factor of volumes = $(\frac{5}{2})^3 = \frac{125}{8}$
$80 \times \frac{125}{8} = 1250$ cm^3
Answer: 1250 cm^3

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6 Perpendicular lines

The diagram shows the straight line L, which passes through the points A and B.
The line M is perpendicular to L and passes through B.
Find the equation of M in the form y = mx + c.

Gradient of L = $\frac{-3-0}{0-4} = \frac{3}{4}$

Perpendicular gradient = $-1 \div \frac{3}{4} = -\frac{4}{3}$

M passes through B(4, 0): $0 = (-\frac{4}{3}) \times 4 + c$, so $c = \frac{16}{3}$

$y = -\frac{4}{3}x + \frac{16}{3}$

Answer: $y = -\frac{4}{3}x + \frac{16}{3}$

7 Squares, cubes and prime factors

Find the smallest positive integer k such that 4116 × k is a square number.

$4116 = 2^2 \times 3 \times 7^3$
For a square number every power must be a multiple of 2
Multiply by 3 × 7 to get $2^2 \times 3^2 \times 7^4$
k = 21
Answer: k = 21

8 Geometric sequences

The 2nd term of a geometric sequence is 32.
The 5th term of the sequence is 256.
(a) Work out the common ratio.
(b) Work out the first term.

(a) The common ratio r satisfies $r^3 = 256 \div 32 = 8$
$r = \sqrt[3]{8} = 2$
(b) The 2nd term is $a \times r^1$: $a \times 2 = 32$
$a = 32 \div 2 = 16$
Answer: (a) r = 2 (b) 16

9 Frustum of a cone

A cone is cut parallel to its base to make a smaller cone and a frustum.
The frustum has height 12 cm, top radius 4 cm and base radius 10 cm.
Work out the volume of the frustum. Give your answer in terms of π.

Let the height of the small cone removed be h cm. The cones are similar, so $\frac{h}{h+12} = \frac{4}{10}$

10 h = 4(h + 12), so 6 h = 48 and h = 8
Large cone: $\frac{1}{3} \times \pi \times 10^2 \times 20 = \frac{2000}{3}\pi$
Small cone: $\frac{1}{3} \times \pi \times 4^2 \times 8 = \frac{128}{3}\pi$
Frustum = $\frac{2000}{3}\pi - \frac{128}{3}\pi = 624\pi$ cm^3
Answer: 624π cm^3

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10 Ratio and mixtures

Alloy A holds copper and tin in the ratio 4 : 5.
Alloy B holds copper and tin in the ratio 6 : 6.
18 kg from alloy A is mixed with some of alloy B.
The ratio of copper to tin in the mixture is 16 : 17.
Work out how many kg were taken from alloy B.

Let y kg be taken from B

copper: $8 + \frac{6y}{12}$

tin: $10 + \frac{6y}{12}$

Ratio 16 : 17, so $17(8 + \frac{6y}{12}) = 16(10 + \frac{6y}{12})$

$(\frac{1}{2})y = 24$, so $y = 48$

Answer: 48 kg