Day 307 **S Tier 10-a-day: Model Solutions**

1 Stratified sampling

The table shows the number of members in each of four groups.
(a) A stratified sample of 28 is taken. Work out how many come from Group C.
(b) A different stratified sample has 12 from Group D. Work out the size of this sample.

Total = 250 + 550 + 300 + 300 = 1400

(a) Group C: $\frac{300}{1400} \times 28 = 6$

(b) Fraction sampled = $12 \div 300 = \frac{1}{25}$, so sample size = $\frac{1}{25} \times 1400 = 56$

Answer: (a) 6 (b) 56

2 Fraction problems

A bag contains marbles.
$\frac{2}{5}$ of the marbles are yellow and $\frac{1}{4}$ of the marbles are black.
The rest of the marbles, 42 of them, are white.
How many black marbles are there?

Fraction that are white = $1 - \frac{2}{5} - \frac{1}{4} = \frac{7}{20}$

$\frac{7}{20}$ of the total is 42

Total = $42 \div \frac{7}{20} = 120$

Black marbles = $\frac{1}{4}$ of 120 = 30

Answer: 30

3 Equations with indices

Solve $3^{2x-2} = 81^{x+3}$

Write both sides as powers of 3: $(3^1)^{2x-2} = (3^4)^{x+3}$
$3^{2x-2} = 3^{4x+12}$
Equate the powers: $2x - 2 = 4x + 12$
$-2x = 14$
$x = -7$
Answer: $x = -7$

4 Multiplying and dividing surds

Simplify fully $\frac{4\sqrt{45}}{3\sqrt{5}}$

$\sqrt{45} = 3\sqrt{5}$
$\frac{4\sqrt{45}}{3\sqrt{5}} = \frac{12\sqrt{5}}{3\sqrt{5}}$
$= 4$
Answer: 4

5 Angles in polygons

Each interior angle of a regular polygon is 8 times the size of each exterior angle.
Work out the number of sides of the polygon.

Let each exterior angle be e. Then each interior angle is 8e.
Interior angle + exterior angle = 180°, so 8e + e = 180°
e = 180° ÷ 9 = 20°
Number of sides = 360° ÷ 20° = 18
Answer: 18

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6 Gradient of a chord

A and B are points on the curve $y = x^3 + 4x$.
The x coordinate of A is 3 and the x coordinate of B is 5.
Work out the gradient of the line AB.

At A: y = 39. At B: y = 145
A is (3, 39) and B is (5, 145)
Gradient = $\frac{145 - 39}{5 - 3} = \frac{106}{2} = 53$

Answer: 53

7 Histograms with an unmarked scale

The histogram shows the masses of parcels delivered by a courier.
360 of them had a mass between 20 and 40 kg.
(a) How many had a mass between 60 and 70 kg?
(b) How many parcels are there in total?

Frequency is proportional to the area of a bar
20 to 40: bar is 6 squares high and 20 wide, area 120, which represents 360
So 1 unit of area represents 360 ÷ 120 = 3
(a) 60 to 70: 1 × 10 = 10 units of area, so 10 × 3 = 30
(b) Total area = 50 + 120 + 120 + 10 = 300, so total = 300 × 3 = 900
Answer: (a) 30 (b) 900

8 Reverse percentage growth and decay

A car loses 10% of its value each year.
After 4 years it is worth £7,217.10.
Work out its value when it was new. (Calculator)

Yearly multiplier = 0.9
Original × 0.9^4 = £7,217.10
Original = £7,217.10 ÷ 0.9^4 = £11,000
Answer: £11,000

9 Area and quadratic equations

The diagram shows a rectangle of length (3x − 1) cm, width (x − 3) cm and area 28 cm^2.
(a) Show that $3x^2 - 10x - 25 = 0$.
(b) Work out the perimeter of the rectangle.

(a) (3x − 1)(x − 3) = 28
Expand: $3x^2 - 10x + 3 = 28$, so $3x^2 - 10x - 25 = 0$
(b) Factorise: (x − 5)(3x + 5) = 0
x = 5 ($x = -\frac{5}{3}$ is not possible as a length would be negative)
Length = 14 cm and width = 2 cm
Perimeter = 2 × (14 + 2) = 32 cm
Answer: (b) 32 cm

10 Prime factors and indices

Write $10^5 \times 15^4$ as a product of powers of prime numbers.

10 = 2 × 5, 15 = 3 × 5
$10^5 = 2^5 \times 5^5$
$15^4 = 3^4 \times 5^4$
Add the powers of each prime: $2^5 \times 3^4 \times 5^9$
Answer: $2^5 \times 3^4 \times 5^9$