Day 306 **S Tier 10-a-day: Model Solutions**

1 Arithmetic sequences

The 6th term of an arithmetic sequence is 49.
The 9th term of the sequence is 76.
(a) Find an expression for the nth term of the sequence.
(b) Is 130 a term of the sequence? You must show your working.

(a) Common difference = (76 − 49) ÷ (9 − 6) = 9
nth term = 9n + c. For n = 6: 9 × 6 + c = 49, so c = −5
nth term = 9n − 5
(b) Solve 9n − 5 = 130: 9n = 135, so n = 15
n is a positive whole number, so 130 is the 15th term
Answer: (a) 9n − 5 (b) Yes, it is the 15th term

2 Ordering numbers

Write these numbers in order of size, starting with the smallest.

83%, $\frac{8}{15}$, $\frac{5}{8}$, $\frac{2}{3}$

Convert each to a decimal: 83% = 0.83, $\frac{8}{15}$ = 0.533, $\frac{5}{8}$ = 0.625, $\frac{2}{3}$ = 0.667

In order: $\frac{8}{15}$, $\frac{5}{8}$, $\frac{2}{3}$, 83%

Answer: $\frac{8}{15}$, $\frac{5}{8}$, $\frac{2}{3}$, 83%

3 Averages with algebra

Here are five numbers: 2x + 2, 2, 5, 2x + 6, 3x.
The mean of the five numbers is 10.
(a) Work out the value of x.
(b) Work out the median of the five numbers.

(a) Total = (2x + 2) + (2) + (5) + (2x + 6) + (3x) = 10 × 5 = 50
7x + 15 = 50, so 7x = 35 and x = 5
The numbers are 12, 2, 5, 16, 15
(b) In order: 2, 5, 12, 15, 16, so the median is 12
Answer: (a) x = 5 (b) 12

4 Sine rule

The diagram shows triangle ABC. (Calculator)
Work out the length of AC. Give your answer correct to 1 decimal place.
The diagram is not drawn accurately.

Use the sine rule: $\frac{AC}{\sin B} = \frac{BC}{\sin A}$

$\frac{x}{\sin 82°} = \frac{6}{\sin 68°}$

$x = \frac{6 \times \sin 82°}{\sin 68°} = 6.4082$

x = 6.4 cm (1 d.p.)
Answer: AC = 6.4 cm

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5 Direct proportion formulae

A is directly proportional to the square root of r.
When r = 4, A = 16.
(a) Find a formula for A in terms of r.
(b) Work out the value of A when r = 16.

(a) $A = k\sqrt{r}$
$16 = k \times 2$, so $k = 16 \div 2 = 8$
$A = 8\sqrt{r}$
(b) $A = 8 \times 4 = 32$
Answer: (a) $A = 8\sqrt{r}$ (b) 32

6 Iteration: rearranging equations

(a) Show that the equation $x^3 + x - 14 = 0$ has a solution between x = 2 and x = 3.
(b) Show that the equation can be rearranged to give $x = \sqrt[3]{14 - x}$

(a) Let $f(x) = x^3 + x - 14$. $f(2) = 2^3 + 2 - 14 = -4$ and $f(3) = 3^3 + 3 - 14 = 16$
f(2) is negative and f(3) is positive. There is a change of sign, so there is a solution between 2 and 3
(b) Rearrange $x^3 + x - 14 = 0$: $x^3 = 14 - x$
Take the cube root of both sides: $x = \sqrt[3]{14 - x}$
Answer: Shown

7 Bearings with the sine and cosine rules

A plane leaves an airport P and travels 44 km on a bearing of 045° to Q. (Calculator)
It then travels 104 km on a bearing of 015° to R.
(a) Work out the distance PR.
(b) Work out the bearing of R from P.
Give distances to 1 decimal place and the bearing to the nearest degree.

(a) The bearing of P from Q is 045° + 180° = 225°
Angle PQR = 150° (the difference between the bearings 225° and 015°)
$PR^2 = 44^2 + 104^2 - 2 \times 44 \times 104 \times \cos 150° = 20678$, so PR = 143.8 km
(b) $\cos(QPR) = \frac{44^2 + PR^2 - 104^2}{2 \times 44 \times PR}$, so angle QPR = 21.2°
R is anticlockwise of the direction PQ, so the bearing is 045° − 21° = 024°
Answer: (a) 143.8 km (b) 024°

8 Profit and loss

A retailer buys a guitar and adds a 30% mark-up to get the selling price.
In a sale the selling price is reduced by 15% to £430.95.
Work out how much the shop paid for a guitar.

Overall multiplier = 1.3 × 0.85 = 1.105
Cost = £430.95 ÷ 1.105
= £390
Answer: £390

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9 Inverse proportion

y is inversely proportional to the square root of x.
When x = 25, y = 4.
Find the positive value of x when y = 5.

$y = \frac{k}{\sqrt{x}}$

$4 = \frac{k}{5}$, so $k = 4 \times 5 = 20$

$5 = \frac{20}{\sqrt{x}}$, so $\sqrt{x} = \frac{20}{5} = 4$

$x = 4^2 = 16$

Answer: x = 16

10 Counting factors

$N = 2 \times 3^4 \times 7$
How many factors does N have?
Include 1 and N.

Add 1 to each power and multiply
$(1 + 1) \times (4 + 1) \times (1 + 1) = 2 \times 5 \times 2 = 20$

Answer: 20