Day 305 **S Tier 10-a-day: Model Solutions**

1 Factorising fully

Factorise fully $2x^3 - 50x$

Take out the common factor 2x: $2x(x^2 - 25)$

$x^2 - 25$ is a difference of two squares: $(x - 5)(x + 5)$

Answer: $2x(x - 5)(x + 5)$

2 Error intervals

The volume of juice is 92 litres, truncated to a whole number.
Write down the error interval for the volume of juice, using v.

Truncating cuts off the later digits without rounding.
The smallest possible value is 92 and the value must be less than 93
$92 \leq v < 93$

Answer: $92 \leq v < 93$

3 Circle theorems: angle at the centre

A, B and C are points on a circle, centre O. Angle ACB = 23°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle OAB.
The diagram is not drawn accurately.

(a) The angle at the centre is twice the angle at the circumference: 2 × 23 = 46°
(b) OA = OB (radii), so triangle OAB is isosceles
Angle OAB = (180 − 46) ÷ 2 = 67°

Answer: (a) 46° (b) 67°

4 Completing the square

(a) Write $4x^2 - 40x + 108$ in the form $a(x + p)^2 + q$
(b) Write down the coordinates of the turning point of the curve $y = 4x^2 - 40x + 108$

(a) Take out the factor 4 from the x terms: $4(x^2 - 10x) + 108$
$x^2 - 10x = (x - 5)^2 - 25$, so $4x^2 - 40x + 108 = 4[(x - 5)^2 - 25] + 108$
$= 4(x - 5)^2 + 8$
(b) The minimum is when $x - 5 = 0$, so x = 5 and y = 8

Answer: (a) $4(x - 5)^2 + 8$ (b) (5, 8)

5 Cosine rule and area of a triangle

The diagram shows triangle ABC. (Calculator)
(a) Work out angle ACB, marked θ, to 1 decimal place.
(b) Work out the area of triangle ABC to 3 significant figures.
The diagram is not drawn accurately.

(a) $\cos\theta = \frac{15^2 + 9^2 - 8^2}{2 \times 15 \times 9} = \frac{242}{270}$

$\theta = 26.3°$ (1 d.p.)

(b) Area $= \frac{1}{2} \times 15 \times 9 \times \sin 26.3°$

$= 29.9\ cm^2$ (3 s.f.)

Answer: (a) 26.3° (b) 29.9 cm^2

6 Prime factors and indices

Write $12^2 \times 20^2$ as a product of powers of prime numbers.

$12 = 2^2 \times 3$, $20 = 2^2 \times 5$

$12^2 = 2^4 \times 3^2$

$20^2 = 2^4 \times 5^2$

Add the powers of each prime: $2^8 \times 3^2 \times 5^2$

Answer: $2^8 \times 3^2 \times 5^2$

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7 Pressure

A metal crate in the shape of a cuboid weighs 1140 N.
Its smallest face has area 600 cm^2. Its largest face has area 1,200 cm^2.
Pressure = force ÷ area
Work out (a) the greatest pressure and (b) the least pressure the metal crate can exert on the ground, in N/m^2.

1 m^2 = 10 000 cm^2, so 600 cm^2 = 0.06 m^2 and 1,200 cm^2 = 0.12 m^2
(a) Greatest pressure is on the smallest face: 1140 ÷ 0.06 = 19,000 N/m^2
(b) Least pressure is on the largest face: 1140 ÷ 0.12 = 9,500 N/m^2
Answer: (a) 19,000 N/m^2 (b) 9,500 N/m^2

8 Equations with algebraic fractions

Solve $\frac{2}{x+5} - \frac{6}{x+2} = \frac{1}{3}$

Multiply every term by $(x + 5)(x + 2)$: $2(x + 2) - 6(x + 5) = \frac{1}{3} \times (x + 5)(x + 2)$
Multiply by 3 to clear the fraction: $6(x + 2) - 18(x + 5) = (x + 5)(x + 2)$
Expand and rearrange so that one side is 0: $x^2 + 19x + 88 = 0$
Factorise: $(x + 8)(x + 11) = 0$
$x = -11$ or $x = -8$
Answer: $x = -11$ or $x = -8$

9 Ratio and mixtures

Tin A holds red paint and white paint in the ratio 5 : 2.
Tin B holds red paint and white paint in the ratio 2 : 5.
7 litres from tin A and 56 litres from tin B are mixed.
Work out the ratio of red paint to white paint in the mixture.

From A: red paint = $7 \times \frac{5}{7}$ = 5 litres, white paint = 2 litres

From B: red paint = $56 \times \frac{2}{7}$ = 16 litres, white paint = 40 litres

red paint: 5 + 16 = 21 and white paint: 2 + 40 = 42
21 : 42 = 1 : 2
Answer: 1 : 2

10 Loci and regions

ABCD is a rectangle with AB = 9 cm and AD = 7 cm.
A point P inside the rectangle is such that AP ≤ 6 cm and P is nearer to AB than to AD.
Work out the area of the region in which P can lie. Give your answer in terms of π.

AP ≤ 6 cm: P is inside the quarter circle of radius 6 cm centred at A (it fits inside the rectangle)
Nearer to AB than AD: P is on the AB side of the angle bisector of angle DAB, which is the line through A at 45° to AB
The bisector cuts the quarter circle into two equal halves
Area = $\frac{1}{2} \times \frac{1}{4} \times \pi \times 6^2 = \frac{9}{2}\pi$ cm^2
Answer: $\frac{9}{2}\pi$ cm^2