Day 304 **S Tier 10-a-day: Model Solutions**

1 Standard form problems

A factory makes 3.9×10^2 bottles each day.
It runs for 8×10^3 days.
How many bottles does it make?
Give your answer in standard form.

Total = $(3.9 \times 10^2) \times (8 \times 10^3)$
$3.9 \times 8 = 31.2$ and $10^2 \times 10^3 = 10^5$
$= 3.12 \times 10^6$
Answer: 3.12×10^6

2 Angles with algebra

In triangle PQR, side QR is extended to S.
Angle QPR = $(2x - 10)°$
Angle PQR = $(2x + 25)°$
Angle PRS = $(2x + 51)°$
(a) Work out the value of x.
(b) Work out the size of angle PRQ.

(a) The exterior angle of a triangle equals the sum of the two opposite interior angles
$2x + 51 = 4x + 15$
$2x = 36$
$x = 18$
(b) Angle PRS = 87° so angle PRQ = $180° - 87° = 93°$
Answer: (a) x = 18 (b) 93°

3 Rational and irrational numbers

Here are six numbers.

$\sqrt{49}$, $(1 + \sqrt{2})^2$, $\frac{1}{\sqrt{3}}$, $0.\dot{4}$, $(\sqrt{3})^2$, $(\sqrt{6} + 2)(\sqrt{6} - 2)$

Write down the rational numbers.

Simplify each number:

$\sqrt{49} = 7$; $(1 + \sqrt{2})^2 = 3 + 2\sqrt{2}$; $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$0.\dot{4} = \frac{4}{9}$; $(\sqrt{3})^2 = 3$; $(\sqrt{6} + 2)(\sqrt{6} - 2) = 6 - 4 = 2$

Answer: $\sqrt{49}$, $0.\dot{4}$, $(\sqrt{3})^2$, $(\sqrt{6} + 2)(\sqrt{6} - 2)$

4 Distance-time graphs

The graph shows Grace's cycle ride to a café and back.
(a) Work out Grace's speed on the way back.
(b) Work out Grace's average speed for the whole ride, including the stop.

(a) Return: 15 km in 1.5 hours, so speed = $15 \div 1.5 = 10$ km/h
(b) Total distance = $2 \times 15 = 30$ km, total time = 4 hours
Average speed = $30 \div 4 = 7.5$ km/h
Answer: (a) 10 km/h (b) 7.5 km/h

5 Speed-time graphs

The graph shows the speed of a train between two stations.
(a) Work out the acceleration in the first 6 seconds.
(b) Work out the total distance.

(a) Acceleration = change in speed ÷ time = $12 \div 6 = 2$ m/s^2
(b) Distance = area under the graph, split into two triangles and a rectangle
$\frac{1}{2} \times 6 \times 12 + 12 \times 15 + \frac{1}{2} \times 4 \times 12 = 36 + 180 + 24$
$= 240$
Answer: (a) 2 m/s^2 (b) 240 m

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6 Reverse percentages

The price of a bike is increased by 25%.
Later the new price is reduced by 40% in a sale.
The sale price is £82.50.
Work out the price before the increase. (Calculator)

Multipliers: 1.25 and 0.6
Overall multiplier = 1.25 × 0.6 = 0.75
Original × 0.75 = £82.50
Original = £82.50 ÷ 0.75 = £110
Answer: £110

7 Geometric sequences

The 2nd term of a geometric sequence is -24.
The 5th term of the sequence is 3.
(a) Work out the common ratio.
(b) Work out the first term.

(a) The common ratio r satisfies $r^3 = 3 \div (-24) = -\frac{1}{8}$

$r = \sqrt[3]{-\frac{1}{8}} = -\frac{1}{2}$

(b) The 2nd term is $a \times r^1$: $a \times (-\frac{1}{2}) = -24$

$a = -24 \div (-\frac{1}{2}) = 48$

Answer: (a) $r = -\frac{1}{2}$ (b) 48

8 Solving trigonometric equations

Solve $2 \sin x = -\sqrt{3}$ for $0° \leq x \leq 360°$.
Do not use a calculator.

$\sin x = -\frac{\sqrt{3}}{2}$

The related acute angle satisfies $\sin \theta = \frac{\sqrt{3}}{2}$, so $\theta = 60°$

sine is negative in the 3rd and 4th quadrants, so $x = 180° + 60°$ and $x = 360° - 60°$
$x = 240°$ and $x = 300°$
Answer: x = 240° or x = 300°

9 Rationalising the denominator

Write $\frac{1+\sqrt{3}}{2-\sqrt{3}}$ in the form $p + q\sqrt{3}$
where p and q are rational numbers.

Multiply top and bottom by $(2 + \sqrt{3})$:
Numerator: $(1 + \sqrt{3})(2 + \sqrt{3}) = 5 + 3\sqrt{3}$
Denominator: $4 - 3 = 1$
$= 5 + 3\sqrt{3}$
Answer: $5 + 3\sqrt{3}$

10 3D Pythagoras and trigonometry: cuboids

The diagram shows a cuboid ABCDEFGH. (Calculator)
(a) Work out the length of the diagonal AG.
(b) Work out the angle between AG and the base ABCD, to 1 decimal place.
The diagram is not drawn accurately.

(a) $AC^2 = AB^2 + BC^2 = 20^2 + 5^2 = 425$
$AG^2 = AC^2 + CG^2 = 425 + 4^2 = 441$
AG = 21 cm

(b) The angle is GAC. $\tan(GAC) = \frac{CG}{AC} = \frac{4}{\sqrt{425}}$

angle GAC = 11.0° (1 d.p.)
Answer: (a) 21 cm (b) 11.0°