Day 303 **S Tier 10-a-day: Model Solutions**

1 Equation of a circle

A circle has centre C(1, 1) and passes through the point P(−2, 6).
Find an equation of the circle.

$r^2 = CP^2 = (-2 - 1)^2 + (6 - 1)^2$
$= 9 + 25 = 34$
$(x - 1)^2 + (y - 1)^2 = 34$
Answer: $(x - 1)^2 + (y - 1)^2 = 34$

2 Mean of a changing set

The mean of 11 numbers is 78.
One number is removed and the mean of the remaining 10 numbers is 81.2.
Work out the number that was removed.

Total of the 11 numbers = 11 × 78 = 858
Total of the 10 remaining = 10 × 81.2 = 812
Number removed = 858 − 812 = 46
Answer: 46

3 Rationalising the denominator

Write $\frac{10}{2 + \sqrt{2}}$ in the form $p + q\sqrt{2}$
where p and q are integers.

Multiply top and bottom by $(2 - \sqrt{2})$:
$\frac{10}{2 + \sqrt{2}} = \frac{10(2 - \sqrt{2})}{(2 + \sqrt{2})(2 - \sqrt{2})}$
Denominator: 4 − 2 = 2
$= 10 - 5\sqrt{2}$
Answer: $10 - 5\sqrt{2}$

4 Pressure

A stone block in the shape of a cuboid weighs 630 N.
Its smallest face has area 800 cm^2. Its largest face has area 1,500 cm^2.
Pressure = force ÷ area
Work out (a) the greatest pressure and (b) the least pressure the stone block can exert on the ground, in N/m^2.

1 m^2 = 10 000 cm^2, so 800 cm^2 = 0.08 m^2 and 1,500 cm^2 = 0.15 m^2
(a) Greatest pressure is on the smallest face: 630 ÷ 0.08 = 7,875 N/m^2
(b) Least pressure is on the largest face: 630 ÷ 0.15 = 4,200 N/m^2
Answer: (a) 7,875 N/m^2 (b) 4,200 N/m^2

5 Laws of indices

Given that $\frac{27^5 \times 27^2}{9} = 3^n$,
find the value of n.

$27 = 3^3$, $9 = 3^2$
Numerator: $3^{15} \times 3^6 = 3^{21}$
Denominator: 3^2
Subtract the powers: 21 − 2 = 19
Answer: n = 19

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6 Equation of a curve from a sketch

The diagram shows a sketch of a quadratic curve with a turning point at (4, −1).
The curve crosses the y-axis at (0, 31).
Find an equation of the curve.
Give your answer in the form $y = ax^2 + bx + c$.

The turning point is (4, −1), so $y = a(x - 4)^2 - 1$
When x = 0, y = 31: $a \times 16 - 1 = 31$, so a = 2
$y = 2(x - 4)^2 - 1$
Expand: $y = 2x^2 - 16x + 31$
Answer: $y = 2x^2 - 16x + 31$

7 Volume and surface area of composite solids

A solid is made from a cone of height 5 cm and a hemisphere. Both have radius 8 cm.
Work out the total volume of the solid. Give your answer in terms of π.

Cone: $\frac{1}{3} \times \pi \times 8^2 \times 5 = \frac{320}{3}\pi$

Hemisphere: $\frac{2}{3} \times \pi \times 8^3 = \frac{1024}{3}\pi$

Total = $\frac{320}{3}\pi + \frac{1024}{3}\pi = 448\pi$ cm^3

Answer: 448π cm^3

8 Tangent to a circle

A circle has equation $x^2 + y^2 = 25$.
The point P(4, 3) lies on the circle.
(a) Find the equation of the tangent to the circle at P. Give your answer in the form ax + by = c, where a, b and c are integers.
(b) The tangent meets the x-axis at the point T. Write down the coordinates of T.

(a) The gradient of the radius OP is $\frac{3}{4}$

The tangent is perpendicular to the radius, so its gradient is $-1 \div \frac{3}{4} = -\frac{4}{3}$

$y - 3 = (-\frac{4}{3})(x - 4)$
Multiply out and rearrange: 4x + 3y = 25
(b) On the x-axis y = 0, so 4x = 25
$x = \frac{25}{4}$, so T is $(\frac{25}{4}, 0)$

Answer: (a) 4x + 3y = 25 (b) T = $(\frac{25}{4}, 0)$

9 Surds with unknowns

Given that $(a + \sqrt{6})^2 = 10 + 4\sqrt{6}$,
where a is a positive integer, find the value of a.

$(a + \sqrt{6})^2 = a^2 + 2a\sqrt{6} + 6$
Compare the $\sqrt{6}$ terms: 2a = 4, so a = 2
Check the integer terms: $a^2 + 6 = 4 + 6 = 10$
Answer: a = 2

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10 Bearings with the sine and cosine rules

A boat leaves a harbour P and travels 37 km on a bearing of 050° to Q. (Calculator)
It then travels 112 km on a bearing of 095° to R.
(a) Work out the distance PR.
(b) Work out the bearing of R from P.
Give distances to 1 decimal place and the bearing to the nearest degree.

(a) The bearing of P from Q is 050° + 180° = 230°
Angle PQR = 135° (the difference between the bearings 230° and 095°)
$PR^2 = 37^2 + 112^2 - 2 \times 37 \times 112 \times \cos 135° = 19774$, so PR = 140.6 km

(b) $\cos(QPR) = \frac{37^2 + PR^2 - 112^2}{2 \times 37 \times PR}$, so angle QPR = 34.3°

R is clockwise of the direction PQ, so the bearing is 050° + 34° = 084°

Answer: (a) 140.6 km (b) 084°