Day 302 **S Tier 10-a-day: Model Solutions**

1 Stratified sampling

The table shows the number of members in each of four groups.
(a) A stratified sample of 23 is taken. Work out how many come from Group C.
(b) A different stratified sample has 6 from Group A. Work out the size of this sample.

Total = 150 + 350 + 450 + 200 = 1150

(a) Group C: $\frac{450}{1150} \times 23 = 9$

(b) Fraction sampled = $6 \div 150 = \frac{1}{25}$, so sample size = $\frac{1}{25} \times 1150 = 46$

Answer: (a) 9 (b) 46

2 Equations with fractions

Solve $\frac{x-5}{2} + \frac{x-4}{4} = -5$

Multiply every term by 4: $2(x - 5) + (x - 4) = -20$
Expand: $3x - 14 = -20$
$3x = -6$
$x = -2$
Answer: $x = -2$

3 Bounds in geometry

A rectangle has length 4.5 cm and width 5.1 cm, both correct to 1 decimal place.
Work out the lower bound of the area of the rectangle.

Each length is given to 1 decimal place, so it can be up to 0.05 cm either side
Length is between 4.45 and 4.55, width is between 5.05 and 5.15
Lower bound of area: use the lower values: $4.45 \times 5.05 = 22.4725$ cm^2
Answer: 22.4725 cm^2

4 Simplifying surds

Write $\sqrt{80} + \sqrt{180} + \sqrt{20} + \sqrt{125}$ in the form $a\sqrt{5}$
where a is an integer.

$\sqrt{80} = 4\sqrt{5}$; $\sqrt{180} = 6\sqrt{5}$; $\sqrt{20} = 2\sqrt{5}$; $\sqrt{125} = 5\sqrt{5}$
$(4 + 6 + 2 + 5)\sqrt{5} = 17\sqrt{5}$
Answer: $17\sqrt{5}$

5 Combined transformations

The point P(6, −3) is rotated 90° anticlockwise about the origin.
The image is then reflected in the line y = x.
(a) Write down the coordinates of the final image of P.
(b) Describe fully the single transformation that is equivalent to these two transformations.

(a) After the first transformation: P' = (3, 6)
After the second transformation: P'' = (6, 3)
(b) Follow a general point (x, y) through both transformations: (x, y) → (−y, x) → (x, −y)
The single mapping is (x, y) → (x, −y)
This is a reflection in the x-axis
Answer: (a) (6, 3) (b) Reflection in the x-axis

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6 Winning a match

Priya and Uma play a match. The first to win 2 chess games wins the match.

The probability that Priya wins any one of the chess games is $\frac{3}{5}$.

Work out the probability that the match needs a third one to decide it.

A third one is needed when each player wins one of the first two

P(Priya wins then loses) = $\frac{3}{5} \times \frac{2}{5} = \frac{6}{25}$

P(Priya loses then wins) = $\frac{2}{5} \times \frac{3}{5} = \frac{6}{25}$

Total = $2 \times \frac{6}{25} = \frac{12}{25}$

Answer: $\frac{12}{25}$

7 Iteration: rearranging equations

(a) Show that the equation $x^3 + x - 7 = 0$ can be rearranged to give $x = \sqrt[3]{7 - x}$ (Calculator)

(b) Use the iteration formula $x_{n+1} = \sqrt[3]{7 - x_n}$ with $x_0 = 1$ to work out x_1 and x_2. Give each answer correct to 3 decimal places.

(a) Rearrange $x^3 + x - 7 = 0$: $x^3 = 7 - x$

Take the cube root of both sides: $x = \sqrt[3]{7 - x}$

(b) $x_1 = \sqrt[3]{7 - 1} = 1.81712...$

$x_2 = \sqrt[3]{7 - x_1} = 1.73057...$

$x_1 = 1.817$ and $x_2 = 1.731$ (3 d.p.)

Answer: (b) $x_1 = 1.817$, $x_2 = 1.731$

8 Exact lengths and areas

A right-angled triangle has base $(5 + \sqrt{7})$ cm and height $(5 - \sqrt{7})$ cm.

Show that the area of the triangle is a rational number.

Work out the area.

Area = $\frac{1}{2} \times (5 + \sqrt{7})(5 - \sqrt{7})$

$(5 + \sqrt{7})(5 - \sqrt{7}) = 25 - 7 = 18$

Area = $\frac{1}{2} \times 18 = 9$

Answer: 9 cm^2

9 Ratio and algebra

In a bag, the ratio of the number of red counters to the number of blue counters is 4 : 3.

Then 10 red counters are removed and 10 blue counters are removed.

The ratio is now 7 : 4.

How many red counters were there at the start?

Let the numbers at the start be 4k and 3k

$(4k - 10) : (3k - 10) = 7 : 4$, so $4(4k - 10) = 7(3k - 10)$

$16k - 40 = 21k - 70$

$-5k = -30$, so $k = 6$

Number of red counters at the start: $4 \times 6 = 24$

Answer: 24

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10 Pythagoras with algebra

ABC is a right-angled triangle with sides (x) cm, (x + 7) cm and (x + 9) cm. The longest side is (x + 9) cm.
(a) Find the value of x.
(b) Work out the area of the triangle.
The diagram is not drawn accurately.

(a) Pythagoras: $x^2 + (x + 7)^2 = (x + 9)^2$
Expand and collect terms: $x^2 - 4x - 32 = 0$
Factorise: $(x - 8)(x + 4) = 0$
$x = 8$ or $x = -4$, and x must be positive, so $x = 8$
(b) The two shorter sides are 8 cm and 15 cm
Area = $\frac{1}{2} \times 8 \times 15 = 60$ cm^2

Answer: (a) x = 8 (b) 60 cm^2