Day 301 **S Tier 10-a-day: Model Solutions**

1 Squares, cubes and prime factors

Find the smallest positive integer k such that 90 × k is a square number.

$90 = 2 \times 3^2 \times 5$

For a square number every power must be a multiple of 2

Multiply by 2×5 to get $2^2 \times 3^2 \times 5^2$

k = 10

Answer: k = 10

2 Column vectors

The vectors $\begin{pmatrix} k \\ 36 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 2k \end{pmatrix}$ are parallel.

k is a positive number.

Work out the value of k.

Parallel vectors: one is a multiple of the other, so the ratio of the components is the same

$\frac{k}{2} = \frac{36}{2k}$

$k \times 2k = 36 \times 2$

$2k^2 = 72$

$k^2 = 36$

k = 6 (k is positive)

Answer: k = 6

3 Iteration

Use the iteration formula $x_{n+1} = \sqrt[3]{2x_n + 2}$ with $x_0 = 4$ (Calculator)

to work out x_1, x_2 and x_3.

Give each answer correct to 3 decimal places.

$x_1 = \sqrt[3]{2 \times 4 + 2} = 2.15443...$

$x_2 = \sqrt[3]{2 \times x_1 + 2} = 1.84778...$

$x_3 = \sqrt[3]{2 \times x_2 + 2} = 1.78585...$

Rounded: $x_1 = 2.154$, $x_2 = 1.848$, $x_3 = 1.786$

Answer: $x_1 = 2.154$, $x_2 = 1.848$, $x_3 = 1.786$

4 Upper and lower bounds

a = 6.3 and b = 2.1,

both correct to 1 decimal place.

Work out the upper bound of a ÷ b.

Give your answer correct to 3 significant figures.

Each value is rounded, so it can be up to 0.05 above or below

$6.25 \leq a < 6.35$; $2.05 \leq b < 2.15$

Biggest answer: biggest a ÷ smallest b = 6.35 ÷ 2.05

= 3.09756... = 3.10 to 3 significant figures

Answer: 3.10

5 Gradient of a chord

A and B are points on the curve $y = -x^2 + x + 5$.

The x coordinate of A is 3 and the x coordinate of B is 5.

Work out the gradient of the line AB.

At A: y = −1. At B: y = −15

A is (3, −1) and B is (5, −15)

Gradient = $\frac{-15 - (-1)}{5 - 3} = \frac{-14}{2} = -7$

Answer: −7

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6 Pressure

A metal crate in the shape of a cuboid weighs 240 N.
Its smallest face has area 600 cm^2. Its largest face has area 1,200 cm^2.
Pressure = force ÷ area
Work out (a) the greatest pressure and (b) the least pressure the metal crate can exert on the ground, in N/m^2.

1 m^2 = 10 000 cm^2, so 600 cm^2 = 0.06 m^2 and 1,200 cm^2 = 0.12 m^2
(a) Greatest pressure is on the smallest face: 240 ÷ 0.06 = 4,000 N/m^2
(b) Least pressure is on the largest face: 240 ÷ 0.12 = 2,000 N/m^2
Answer: (a) 4,000 N/m^2 (b) 2,000 N/m^2

7 Bearings with the sine and cosine rules

A boat leaves a harbour P and travels 55 km on a bearing of 345° to Q. (Calculator)
It then travels 32 km on a bearing of 260° to R.
(a) Work out the distance PR.
(b) Work out the bearing of R from P.
Give distances to 1 decimal place and the bearing to the nearest degree.

(a) The bearing of P from Q is 345° − 180° = 165°
Angle PQR = 95° (the difference between the bearings 165° and 260°)
$PR^2 = 55^2 + 32^2 - 2 \times 55 \times 32 \times \cos 95° = 4355.8$, so PR = 66.0 km
(b) $\cos(QPR) = \frac{55^2 + PR^2 - 32^2}{2 \times 55 \times PR}$, so angle QPR = 28.9°
R is anticlockwise of the direction PQ, so the bearing is 345° − 29° = 316°
Answer: (a) 66.0 km (b) 316°

8 Speed and a moving train

A train travels at a constant speed of 36 km/h.
It takes 25 seconds to cross completely over a bridge which is 120 m long.
Work out the length of the train, in metres.

Speed = 36 km/h = 36,000 ÷ 3600 = 10 m/s
Distance travelled in 25 s = 10 × 25 = 250 m
This is the length of the bridge plus the length of the train
Train = 250 − 120 = 130 m
Answer: 130 m

9 Vectors: intersecting lines

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 2 : 1. N is on OB with ON : NB = 1 : 3. AN and BM intersect at X.
(a) Find BM in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Find OX in terms of $\mathbf{a}$ and $\mathbf{b}$.
The diagram is not drawn accurately.

(a) OM = $\frac{2}{3}\mathbf{a}$, so BM = $-\mathbf{b} + \frac{2}{3}\mathbf{a} = \frac{2}{3}\mathbf{a} - \mathbf{b}$

(b) X is on BM: BX = s BM, so OX = OB + s BM = $\frac{2}{3}s\mathbf{a} + (1 - s)\mathbf{b}$

X is on AN: OX = OA + t AN = OA + t(ON − OA) = $(1 - t)\mathbf{a} + \frac{1}{4}t\mathbf{b}$

Equate: $\frac{2}{3}s = 1 - t$ and $1 - s = \frac{1}{4}t$

Solving: $s = \frac{9}{10}$ and $t = \frac{2}{5}$

OX = $\frac{3}{5}\mathbf{a} + \frac{1}{10}\mathbf{b}$

Answer: (a) $\frac{2}{3}\mathbf{a} - \mathbf{b}$ (b) OX = $\frac{3}{5}\mathbf{a} + \frac{1}{10}\mathbf{b}$

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10 Pythagoras and quadratic equations

The lengths of the sides of a right-angled triangle are $(x - 2)$ cm, $(x - 3)$ cm and $(x - 1)$ cm.
The longest side is $(x - 1)$ cm.
(a) Show that $x^2 - 8x + 12 = 0$
(b) Hence work out the area of the triangle.

(a) Pythagoras: $(x - 2)^2 + (x - 3)^2 = (x - 1)^2$
Expand: $x^2 - 4x + 4 + x^2 - 6x + 9 = x^2 - 2x + 1$
Rearrange: $x^2 - 8x + 12 = 0$
(b) Factorise: $(x - 6)(x - 2) = 0$, so $x = 6$ (the other root, $x = 2$, does not give valid side lengths)
The shorter sides are 4 cm and 3 cm, so area = $\frac{1}{2} \times 4 \times 3 = 6$ cm^2

Answer: (b) 6 cm^2