Day 300 **S Tier 10-a-day: Model Solutions**

1 Simplifying surds

Write $\sqrt{252} - \sqrt{7} + \sqrt{175}$ in the form $a\sqrt{7}$
where a is an integer.

$\sqrt{252} = 6\sqrt{7}$; $\sqrt{175} = 5\sqrt{7}$

$(6 - 1 + 5)\sqrt{7} = 10\sqrt{7}$

Answer: $10\sqrt{7}$

2 Sine rule

The diagram shows triangle ABC. (Calculator)
Work out the length of AC. Give your answer correct to 1 decimal place.
The diagram is not drawn accurately.

Use the sine rule: $\frac{AC}{\sin B} = \frac{BC}{\sin A}$

$\frac{x}{\sin 46°} = \frac{7}{\sin 39°}$

$x = \frac{7 \times \sin 46°}{\sin 39°} = 8.0013$

x = 8.0 cm (1 d.p.)

Answer: AC = 8.0 cm

3 Inverse functions

$f(x) = \frac{7}{x - 4} + 2$

(a) Find $f^{-1}(x)$.

(b) Find the value of $f^{-1}(-5)$.

(a) Let $y = \frac{7}{x - 4} + 2$, so $y - 2 = \frac{7}{x - 4}$

Make x the subject: $x - 4 = \frac{7}{y - 2}$

$x = \frac{7}{y - 2} + 4$

$f^{-1}(x) = \frac{7}{x - 2} + 4$

(b) $f^{-1}(-5) = \frac{7}{-5 - 2} + 4 = 3$

Answer: (a) $f^{-1}(x) = \frac{7}{x - 2} + 4$ (b) 3

4 Distance-time graphs

The graph shows Jamal's cycle ride to a café and back.
(a) Work out Jamal's speed on the way back.
(b) Work out Jamal's average speed for the whole ride, including the stop.

(a) Return: 24 km in 3 hours, so speed = 24 ÷ 3 = 8 km/h
(b) Total distance = 2 × 24 = 48 km, total time = 5 hours
Average speed = 48 ÷ 5 = 9.6 km/h
Answer: (a) 8 km/h (b) 9.6 km/h

5 Expanding three brackets

Expand and simplify $(x + 3)^2(x + 6)$

Expand the first two brackets: $(x + 3)^2 = x^2 + 6x + 9$
Multiply by (x + 6): $x(x^2 + 6x + 9) + 6(x^2 + 6x + 9)$
$= x^3 + 12x^2 + 45x + 54$
Answer: $x^3 + 12x^2 + 45x + 54$

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6 Combined transformations

The point P(−1, 4) is reflected in the y-axis.
The image is then rotated 90° clockwise about the origin.
(a) Write down the coordinates of the final image of P.
(b) Describe fully the single transformation that is equivalent to these two transformations.

(a) After the first transformation: P' = (1, 4)
After the second transformation: P'' = (4, −1)
(b) Follow a general point (x, y) through both transformations: (x, y) → (−x, y) → (y, x)
The single mapping is (x, y) → (y, x)
This is a reflection in the line y = x
Answer: (a) (4, −1) (b) Reflection in the line y = x

7 Speed and a moving train

A train travels at a constant speed of 108 km/h.
It takes 30 seconds to cross completely over a bridge which is 710 m long.
Work out the length of the train, in metres.

Speed = 108 km/h = 108,000 ÷ 3600 = 30 m/s
Distance travelled in 30 s = 30 × 30 = 900 m
This is the length of the bridge plus the length of the train
Train = 900 − 710 = 190 m
Answer: 190 m

8 Iteration: rearranging equations

(a) Show that the equation $x^2 - 2x - 5 = 0$ has a solution between x = 3 and x = 4.
(b) Show that the equation can be rearranged to give $x = 2 + \frac{5}{x}$

(a) Let $f(x) = x^2 - 2x - 5$. $f(3) = 3^2 - 2 \times 3 - 5 = -2$ and $f(4) = 4^2 - 2 \times 4 - 5 = 3$
f(3) is negative and f(4) is positive. There is a change of sign, so there is a solution between 3 and 4
(b) Rearrange $x^2 - 2x - 5 = 0$: $x^2 - 2x = 5$
Factorise: $x(x - 2) = 5$
Divide by x: $x - 2 = \frac{5}{x}$, so $x = 2 + \frac{5}{x}$
Answer: Shown

9 Percentages with an unknown rate

The price of a share is decreased by x%.
The new price is then increased by x%.
The final price is 25% less than the original price.
Work out the value of x.

Let the original price be 1. Multipliers: $(1 + \frac{x}{100})$ and $(1 - \frac{x}{100})$

$(1 + \frac{x}{100})(1 - \frac{x}{100}) = 1 - \frac{x^2}{10000}$

Final price is 25% less, so the multiplier is 0.75: $1 - \frac{x^2}{10000} = 0.75$

$\frac{x^2}{10000} = 0.25$, so $x^2 = 2500$ and x = 50
Answer: x = 50

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10 Frustum of a cone

A cone is cut parallel to its base to make a smaller cone and a frustum.
The frustum has height 21 cm, top radius 4 cm and base radius 16 cm.
Work out the volume of the frustum. Give your answer in terms of π.

Let the height of the small cone removed be h cm. The cones are similar, so $\frac{h}{h+21} = \frac{4}{16}$

16 h = 4(h + 21), so 12 h = 84 and h = 7

Large cone: $\frac{1}{3} \times \pi \times 16^2 \times 28 = \frac{7168}{3}\pi$

Small cone: $\frac{1}{3} \times \pi \times 4^2 \times 7 = \frac{112}{3}\pi$

Frustum = $\frac{7168}{3}\pi - \frac{112}{3}\pi = 2352\pi$ cm^3

Answer: 2352π cm^3