Day 30 **S Tier 10-a-day: Model Solutions**

1 Equations with fractions

Solve $\frac{x+6}{5} + \frac{x-5}{2} = -2$

Multiply every term by 10: $2(x + 6) + 5(x - 5) = -20$
Expand: $7x - 13 = -20$
$7x = -7$
$x = -1$
Answer: $x = -1$

2 Line of best fit

The scatter graph shows distance from town (x) and price (y).
The line of best fit passes through (2, 38) and (6, 36).
(a) Find its equation in the form $y = mx + c$.
(b) Explain what its gradient tells you.

(a) Gradient $m = \frac{36 - 38}{6 - 2} = \frac{-2}{4} = -0.5$
$y = -0.5x + c$: using (2, 38), $38 = -0.5 \times 2 + c$, so $c = 39$
$y = -0.5x + 39$
(b) For each extra kilometre from the town, the house price (in £10 000s) goes down by 0.5
Answer: (a) $y = -0.5x + 39$ (b) For each extra kilometre from the town, the house price (in £10 000s) goes down by 0.5

3 LCM problems

A church bell rings every 4 minutes.
Another bell rings every 6 minutes.
A third bell rings every 21 minutes.
They all ring together at 9:00 am.
How many times do they all ring together between 9:00 am and 3:00 pm, including both 9:00 am and 3:00 pm if they do?

Find the LCM of 4, 6 and 21
$4 = 2^2$, $6 = 2 \times 3$, $21 = 3 \times 7$
LCM $= 2^2 \times 3 \times 7 = 84$
Total time from 9:00 am to 3:00 pm = 360 minutes
$360 \div 84 = 4.29$, so there are 4 more times after the start
Including the start: $4 + 1 = 5$
Answer: 5

4 Weighted percentages

The ratio of juniors to seniors in a school is 3 : 2.
10% of the juniors and 50% of the seniors play a sport.
What percentage of all of these students play a sport?

Imagine 100 students in total: 60 juniors and 40 seniors
10% of 60 = 6 and 50% of 40 = 20
Together: 26 out of 100
$26 \div 100 \times 100 = 26\%$
Answer: 26%

5 Expanding brackets with surds

Expand and simplify $(6 + 2\sqrt{6})(6 + \sqrt{6})$
Give your answer in the form $p + q\sqrt{6}$

$(6 + 2\sqrt{6})(6 + \sqrt{6}) = 36 + 6\sqrt{6} + 12\sqrt{6} + 12$
$= 48 + 18\sqrt{6}$
Answer: $48 + 18\sqrt{6}$

Day 30 **S Tier 10-a-day: Model Solutions**

6 Similar triangles with algebra

In the diagram, DE is parallel to BC.
AD = x cm, DB = (x + 5) cm, DE = 3 cm and BC = 9 cm.
(a) Find the value of x.
(b) Work out the length of AB.
The diagram is not drawn accurately.

(a) Triangles ADE and ABC are similar, so $\frac{AD}{AB} = \frac{DE}{BC}$

$\frac{x}{2x + 5} = \frac{3}{9}$
$9x = 3(2x + 5)$
$9x = 6x + 15$
$3x = 15$
$x = 5$
(b) AB = 2x + 5 = 15 cm
Answer: (a) x = 5 (b) 15 cm

7 Pythagoras and quadratic equations

The lengths of the sides of a right-angled triangle are (x − 5) cm, (x − 4) cm and (x − 3) cm.
The longest side is (x − 3) cm.
(a) Show that $x^2 - 12x + 32 = 0$
(b) Hence work out the area of the triangle.

(a) Pythagoras: $(x - 5)^2 + (x - 4)^2 = (x - 3)^2$
Expand: $x^2 - 10x + 25 + x^2 - 8x + 16 = x^2 - 6x + 9$
Rearrange: $x^2 - 12x + 32 = 0$
(b) Factorise: $(x - 8)(x - 4) = 0$, so x = 8 (the other root, x = 4, does not give valid side lengths)
The shorter sides are 3 cm and 4 cm, so area = $\frac{1}{2} \times 3 \times 4 = 6$ cm^2

Answer: (b) 6 cm^2

8 Vector proof: collinear points

OA = **a** and OB = **b**.
OP = 3**b**, OQ = −2**a** + 13**b** and OR = −6**a** + 33**b**.
(a) Show that P, Q and R lie on a straight line.
(b) Work out the ratio PQ : QR in its simplest form.

(a) PQ = OQ − OP = −2**a** + 10**b**
QR = OR − OQ = −4**a** + 20**b**
QR = 2 PQ, so PQ and QR are parallel, and they share the point Q
So P, Q and R lie on a straight line
(b) PQ : QR = 2 : 4 = 1 : 2
Answer: (b) 1 : 2

9 Inequalities

(a) Solve 6 − 3x ≥ −6
(b) n is an integer. Write down all the values of n that satisfy
−2 < 3n + 1 < 10

(a) Subtract 6: −3x ≥ −12
Divide by −3 and reverse the inequality sign: x ≤ 4
(b) Subtract 1: −3 < 3n < 9
Divide by 3: −1 < n < 3
Integers: 0, 1, 2
Answer: (a) x ≤ 4 (b) 0, 1, 2

Day 30 **S Tier 10-a-day: Model Solutions**

10 Recurring decimals

Write $0.00\dot{3}\dot{9}$ as a fraction in its simplest form.

Let x = 0.003903...

10x = 0.039039...

10,000x = 39.039039...

Subtract: 9990x = 39

$x = \frac{39}{9990} = \frac{13}{3330}$

Answer: $\mathbf{\frac{13}{3330}}$