Day 3 **S Tier 10-a-day: Model Solutions**

1 Time zones

A flight leaves London at 15:50 (London time).
It takes 12 hours 15 minutes.
When it is 12:00 in London it is 04:00 in Los Angeles.
At what time, in Los Angeles time, does the flight land? Say if it is on a different day.

Arrival in London time = 15:50 + 12 h 15 min = 04:05 the next day
Los Angeles is 8 hours behind London, so subtract 8 hours
20:05
Answer: 20:05 the same day

2 Inverse proportion in context

10 painters can paint a hall in 8 days.
After 6 days, 5 of the painters leave. The rest carry on working at the same rate.
How many more days does it take to finish?

Total work = 10 × 8 = 80 painter-days
Work done = 10 × 6 = 60, so work left = 20
Workers left = 10 − 5 = 5
Time = 20 ÷ 5 = 4 days
Answer: 4 days

3 Inequalities

(a) Solve $10 - 5x > 10$
(b) n is an integer. Write down all the values of n that satisfy
$-7 < 3n + 2 < 8$

(a) Subtract 10: $-5x > 0$
Divide by -5 and reverse the inequality sign: $x < 0$
(b) Subtract 2: $-9 < 3n < 6$
Divide by 3: $-3 < n < 2$
Integers: $-2, -1, 0, 1$
Answer: (a) $x < 0$ (b) $-2, -1, 0, 1$

4 Rational and irrational numbers

Here are six numbers.

$\sqrt{2} \times \sqrt{6}$, $\frac{1}{\sqrt{3}}$, $\sqrt[3]{27}$, $\frac{\sqrt{18}}{\sqrt{2}}$, $\frac{\sqrt{20}}{\sqrt{5}}$, $(\sqrt{3})^2$

Write down the rational numbers.

Simplify each number:

$\sqrt{2} \times \sqrt{6} = \sqrt{12} = 2\sqrt{3}$; $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$; $\sqrt[3]{27} = 3$

$\frac{\sqrt{18}}{\sqrt{2}} = \sqrt{9} = 3$; $\frac{\sqrt{20}}{\sqrt{5}} = \sqrt{4} = 2$; $(\sqrt{3})^2 = 3$

Answer: $\sqrt[3]{27}$, $\frac{\sqrt{18}}{\sqrt{2}}$, $\frac{\sqrt{20}}{\sqrt{5}}$, $(\sqrt{3})^2$

5 Gradient of a chord

A and B are points on the curve $y = x^3 - 4x$.
The x coordinate of A is -3 and the x coordinate of B is -2.
Work out the gradient of the line AB.

At A: $y = -15$. At B: $y = 0$
A is $(-3, -15)$ and B is $(-2, 0)$
Gradient = $\frac{0 - (-15)}{-2 - (-3)} = \frac{15}{1} = 15$
Answer: 15

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6 Frustum of a cone

A cone is cut parallel to its base to make a smaller cone and a frustum.
The frustum has height 14 cm, top radius 4 cm and base radius 12 cm.
Work out the volume of the frustum. Give your answer in terms of π.

Let the height of the small cone removed be h cm. The cones are similar, so $\frac{h}{h+14} = \frac{4}{12}$

$12h = 4(h + 14)$, so $8h = 56$ and $h = 7$

Large cone: $\frac{1}{3} \times \pi \times 12^2 \times 21 = 1008\pi$

Small cone: $\frac{1}{3} \times \pi \times 4^2 \times 7 = \frac{112}{3}\pi$

Frustum = $1008\pi - \frac{112}{3}\pi = \frac{2912}{3}\pi$ cm^3

Answer: $\frac{2912}{3}\pi$ cm^3

7 At least one

The probability that a bus is more than 5 minutes late on any morning is $\frac{1}{4}$.

Events on different occasions are independent.
Work out the smallest number of mornings needed so that the probability of this happening at least once is greater than 0.75. (Calculator)

P(at least once in n mornings) = $1 - \left(\frac{3}{4}\right)^n$

n = 4: $1 - \left(\frac{3}{4}\right)^4 = 0.6836$

n = 5: $1 - \left(\frac{3}{4}\right)^5 = 0.7627$

0.7627 is the first value above 0.75
Answer: 5

8 Index equations

Solve $125^{x+2} = 5^{2x+2}$

Write both sides as powers of 5: $125 = 5^3$
Equate the powers: $3x + 6 = 2x + 2$
$x = -4$
Answer: $x = -4$

9 Area of a segment

The circle has centre O and radius 10 cm. A and B are points on the circle and angle AOB = 140°. (Calculator)
Work out the area of the shaded segment. Give your answer correct to 3 significant figures.
The diagram is not drawn accurately.

Area of sector OAB = $\frac{140}{360} \times \pi \times 10^2 = 122.17$ cm^2

Area of triangle OAB = $\frac{1}{2} \times 10 \times 10 \times \sin 140° = 32.139$ cm^2

Segment = sector − triangle = 122.17 − 32.139 = 90.034
= 90.0 cm^2 (3 s.f.)
Answer: 90.0 cm^2

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10 Probability from a frequency tree

In a population of 2,000 people, 10% have a certain disease.
A test gives a positive result for 90% of people who have the disease and for 2% of people who do not.
A person is chosen at random and tests positive.
Work out the probability that they have the disease.

10% of 2,000 = 200 have the disease and 1,800 do not
Flagged and genuine: 200 × 90% = 180
Flagged but not: 1,800 × 2% = 36
Total flagged or positive = 180 + 36 = 216

Probability = 180 ÷ 216 = $\frac{5}{6}$

Answer: $\frac{5}{6}$