Day 299 **S Tier 10-a-day: Model Solutions**

1 Similar shapes

Two shapes are mathematically similar.
The smaller has height 5 cm and area 12 cm^2.
The larger has area 48 cm^2.
Work out the height of the larger shape.

Ratio of areas = 48 : 12 = 4 : 1
Ratio of lengths = 2 : 1
Height = 5 × 2 = 10 cm
Answer: 10 cm

2 Expanding three brackets

Expand and simplify $(x + 1)^2(x + 6)$

Expand the first two brackets: $(x + 1)^2 = x^2 + 2x + 1$
Multiply by $(x + 6)$: $x(x^2 + 2x + 1) + 6(x^2 + 2x + 1)$
$= x^3 + 8x^2 + 13x + 6$
Answer: $x^3 + 8x^2 + 13x + 6$

3 Weighted percentages

The ratio of boys to girls in a school is 2 : 3.
50% of the boys and 85% of the girls walk to school.
What percentage of all of these students walk to school?

Imagine 100 students in total: 40 boys and 60 girls
50% of 40 = 20 and 85% of 60 = 51
Together: 71 out of 100
71 ÷ 100 × 100 = 71%
Answer: 71%

4 Compound interest

Evan invests £5,000.
The account pays compound interest of 6% a year for the first 3 years,
then 3.5% a year for the next 2 years.
Work out the total amount after 5 years, to the nearest penny. (Calculator)

Multipliers: 1.06 for 3 years, then 1.035 for 2 years
£5,000 × 1.06^3 × 1.035^2
= £6,379.23
Answer: £6,379.23

5 Speed-time graphs

The graph shows the speed of a cyclist during 22 seconds.
(a) Work out the acceleration during the first 8 seconds.
(b) Work out the average speed over the whole 22 seconds.

(a) Acceleration = change in speed ÷ time = (16 − 0) ÷ 8 = 2 m/s^2
(b) Distance = area under the graph
Trapezium: $\frac{0 + 16}{2}$ × 8 = 64; rectangle: 16 × 11 = 176; triangle: $\frac{1}{2}$ × 3 × 16 = 24
Total = 264 m, so average speed = 264 ÷ 22 = 12 m/s
Answer: (a) 2 m/s^2 (b) 12 m/s

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6 Rearranging when the subject appears twice

Make u the subject of $v = \frac{7u + 1}{u - 3}$

Multiply by $u - 3$: $v(u - 3) = 7u + 1$
Expand: $uv - 3v = 7u + 1$
Collect the u terms: $uv - 7u = 3v + 1$
Factorise: $u(v - 7) = 3v + 1$
$u = \frac{3v + 1}{v - 7}$

Answer: $u = \frac{3v + 1}{v - 7}$

7 3D problems: pyramids

VABCD is a pyramid on a square base. V is directly above the centre M.
(a) Work out the height VM of the pyramid.
(b) Work out the volume of the pyramid.
The diagram is not drawn accurately.

(a) $AC^2 = 12^2 + 12^2 = 288$, so $AM^2 = (AC \div 2)^2 = 72$
$VM^2 = VA^2 - AM^2 = 9^2 - 72 = 9$
VM = 3 cm

(b) Volume $= \frac{1}{3} \times 12^2 \times 3 = 144$ cm^3

Answer: (a) 3 cm (b) 144 cm^3

8 Probability from a frequency tree

A factory makes 5,000 parts. 2% of them are faulty.
A scanner flags 98% of faulty parts and wrongly flags 4% of parts that are not faulty.
A part is chosen at random from the parts flagged by the scanner.
Work out the probability that this part is faulty.

2% of 5,000 = 100 are faulty and 4,900 are not
Flagged and genuine: 100 × 98% = 98
Flagged but not: 4,900 × 4% = 196
Total flagged or positive = 98 + 196 = 294

Probability = 98 ÷ 294 = $\frac{1}{3}$

Answer: $\frac{1}{3}$

9 Tangent to a circle

A circle has equation $x^2 + y^2 = 65$.
The point P(−7, −4) lies on the circle.
Find an equation of the tangent to the circle at P.
Give your answer in the form ax + by = c, where a, b and c are integers.

The centre is (0, 0). The gradient of the radius to P is $\frac{-4 - 0}{-7 - 0} = \frac{4}{7}$

The tangent is perpendicular to the radius, so its gradient is $-\frac{7}{4}$. Using P: $y + 4 = -\frac{7}{4}(x + 7)$

Multiply by 4: $4(y + 4) = -7(x + 7)$
Expand and rearrange: $7x + 4y = -65$
Answer: $7x + 4y = -65$

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10 Surds with unknowns

Given that $(a + \sqrt{7})(b + \sqrt{7}) = 19 + 8\sqrt{7}$,
where a and b are positive integers with a < b, find the values of a and b.

$(a + \sqrt{7})(b + \sqrt{7}) = ab + (a + b)\sqrt{7} + 7$

Compare the $\sqrt{7}$ terms: $a + b = 8$

Compare the integer terms: $ab + 7 = 19$, so $ab = 12$

Two numbers with sum 8 and product 12 are 2 and 6

Answer: a = 2, b = 6