Day 298 **S Tier 10-a-day: Model Solutions**

1 Inverse proportion in context

12 painters can paint a hall in 20 days.
After 2 days, 4 of the painters leave. The rest carry on working at the same rate.
How many more days does it take to finish?

Total work = 12 × 20 = 240 painter-days
Work done = 12 × 2 = 24, so work left = 216
Workers left = 12 − 4 = 8
Time = 216 ÷ 8 = 27 days
Answer: 27 days

2 Factorising quadratics

Factorise $9x^2 - 27x + 20$

9 × 20 = 180. Two numbers that multiply to 180 and add to −27 are −12 and −15
Split the middle term: $9x^2 - 12x - 15x + 20$
Factorise in pairs: $3x(3x - 4) - 5(3x - 4)$
$= (3x - 5)(3x - 4)$
Answer: $(3x - 5)(3x - 4)$

3 Fraction problems

A bag contains counters.
$\frac{1}{3}$ of the counters are red and $\frac{3}{8}$ of the counters are blue.
The rest of the counters, 28 of them, are green.
How many counters are in the bag?

Fraction that are green = $1 - \frac{1}{3} - \frac{3}{8} = \frac{7}{24}$

$\frac{7}{24}$ of the total is 28

Total = $28 \div \frac{7}{24} = 96$

Answer: 96

4 Recasting solids

A solid cylinder of radius 12 cm and height 13 cm is melted down.
The metal is used to make identical solid spheres of radius 3 cm.
Work out the number of spheres that can be made.

The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.

Volume of cylinder = $\pi \times 12^2 \times 13 = 1872\pi$

Volume of one sphere = $\frac{4}{3} \times \pi \times 3^3 = 36\pi$

Number of spheres = 1872 ÷ 36 = 52
Answer: 52

5 Percentage change in area and volume

The side length of a square is increased by 25%.
Work out the percentage change in the area of the square.

Side length multiplier = 1.25
Area multiplier = $1.25^2 = 1.5625$
1.5625 − 1 = 0.5625, so a 56.25% increase
Answer: 56.25% increase

Day 298 **S Tier 10-a-day: Model Solutions**

6 Fractions removed from a ratio

The ratio of men to women people at a meeting is 8 : 9.

Then $\frac{3}{4}$ of the men and $\frac{1}{6}$ of the women are taken out.

Work out the ratio of men to women now.

Left: $\frac{1}{4} \times 8 = 2$ of the men part, $\frac{5}{6} \times 9 = \frac{15}{2}$ of the women part

$2 : \frac{15}{2}$

Multiply by 2: 4 : 15

Answer: 4 : 15

7 Area of a segment

The circle has centre O and radius 9 cm. A and B are points on the circle and angle AOB = 110°. (Calculator)
Work out the area of the shaded segment. Give your answer correct to 3 significant figures.
The diagram is not drawn accurately.

Area of sector OAB = $\frac{110}{360} \times \pi \times 9^2 = 77.754$ cm^2

Area of triangle OAB = $\frac{1}{2} \times 9 \times 9 \times \sin 110° = 38.058$ cm^2

Segment = sector − triangle = 77.754 − 38.058 = 39.697

= 39.7 cm^2 (3 s.f.)

Answer: 39.7 cm^2

8 Tangent to a circle

A circle has equation $(x - 4)^2 + (y + 3)^2 = 50$.
The point P(−1, −8) lies on the circle.
Find an equation of the tangent to the circle at P.
Give your answer in the form ax + by = c, where a, b and c are integers.

The centre is (4, −3). The gradient of the radius to P is $\frac{-8 - (-3)}{-1 - 4} = 1$

The tangent is perpendicular to the radius, so its gradient is −1. Using P: y + 8 = −(x + 1)
Expand and rearrange: x + y = −9

Answer: x + y = −9

9 Nested surds

Show that $\sqrt{14 + 6\sqrt{5}} = 3 + \sqrt{5}$

Square the right-hand side: $(3 + \sqrt{5})^2 = 9 + 6\sqrt{5} + 5$

$= 14 + 6\sqrt{5}$

So $\sqrt{14 + 6\sqrt{5}} = 3 + \sqrt{5}$

Answer: $\sqrt{14 + 6\sqrt{5}} = 3 + \sqrt{5}$

10 Tangent to a circle: proof

Show that the line y = −3x + 10 is a tangent to the circle $x^2 + y^2 = 10$.
Find the coordinates of the point where the line touches the circle.

Substitute y = −3x + 10 into $x^2 + y^2 = 10$: $x^2 + (-3x + 10)^2 = 10$

Expand: $10x^2 - 60x + 90 = 0$

$b^2 - 4ac = (-60)^2 - 4 \times 10 \times 90 = 3600 - 3600 = 0$

$b^2 - 4ac = 0$, so there is exactly one point of intersection: the line is a tangent

$x = \frac{-(-60)}{2 \times 10} = 3$, and $y = -3 \times 3 + 10 = 1$. The point is (3, 1)

Answer: (3, 1)