Day 297 **S Tier 10-a-day: Model Solutions**

1 Averages with algebra

Here are five numbers: $2x + 6$, $2x - 1$, $x + 1$, $x + 6$, $3x - 1$.
The mean of the five numbers is 13.
(a) Work out the value of x.
(b) Work out the range of the five numbers.

(a) Total = $(2x + 6) + (2x - 1) + (x + 1) + (x + 6) + (3x - 1) = 13 \times 5 = 65$
$9x + 11 = 65$, so $9x = 54$ and $x = 6$
The numbers are 18, 11, 7, 12, 17
(b) Range = $18 - 7 = 11$
Answer: (a) x = 6 (b) 11

2 Rationalising the denominator

Rationalise the denominator of $\frac{24}{\sqrt{6}}$
Give your answer in its simplest form.

Multiply top and bottom by $\sqrt{6}$:
$\frac{24}{\sqrt{6}} = \frac{24\sqrt{6}}{6}$
$= 4\sqrt{6}$
Answer: $4\sqrt{6}$

3 Pythagoras and quadratic equations

The lengths of the sides of a right-angled triangle are x cm, (x + 7) cm and (x + 9) cm.
The longest side is (x + 9) cm.
(a) Show that $x^2 - 4x - 32 = 0$
(b) Hence work out the area of the triangle.

(a) Pythagoras: $x^2 + (x + 7)^2 = (x + 9)^2$
Expand: $x^2 + x^2 + 14x + 49 = x^2 + 18x + 81$
Rearrange: $x^2 - 4x - 32 = 0$
(b) Factorise: $(x - 8)(x + 4) = 0$, so $x = 8$ (the other root, $x = -4$, does not give valid side lengths)
The shorter sides are 8 cm and 15 cm, so area = $\frac{1}{2} \times 8 \times 15 = 60$ cm^2
Answer: (b) 60 cm^2

4 Cosine rule and area of a triangle

The diagram shows triangle ABC. (Calculator)
(a) Work out angle ACB, marked θ, to 1 decimal place.
(b) Work out the area of triangle ABC to 3 significant figures.
The diagram is not drawn accurately.

(a) $\cos\theta = \frac{11^2 + 10^2 - 16^2}{2 \times 11 \times 10} = \frac{-35}{220}$
$\theta = 99.2°$ (1 d.p.)
(b) Area = $\frac{1}{2} \times 11 \times 10 \times \sin 99.2°$
$= 54.3$ cm^2 (3 s.f.)
Answer: (a) 99.2° (b) 54.3 cm^2

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5 Simple and compound interest

Omar invests £1,000 for 5 years at 2% per year simple interest.
Jess invests £1,000 for 5 years at 2% per year compound interest.
Work out how much more interest Jess earns than Omar.
Give your answer to the nearest penny. (Calculator)

Simple interest: £1,000 × 2% × 5 = £100
Compound: £1,000 × 1.02^5 = £1,104.08
Compound interest = £1,104.08 − £1,000 = £104.08
Difference = £104.08 − £100 = £4.08
Answer: £4.08

6 Average speed

A bus travels the first half of a journey at 30 km/h.
It travels the second half of the same journey at 45 km/h.
Work out the average speed for the whole journey.

Let each half of the journey be 90 km (a common multiple of 30 and 45)
Time for first half = 90 ÷ 30 = 3 hours
Time for second half = 90 ÷ 45 = 2 hours
Average speed = total distance ÷ total time = 180 ÷ 5 = 36 km/h
Answer: 36 km/h

7 Trigonometric equations

Solve $4 \sin x° - 6 = -8$ for $0 \leq x \leq 360$

Rearrange: $4 \sin x° = -2$, so $\sin x° = -\frac{1}{2}$

The acute angle with sine $\frac{1}{2}$ is 30°; sine is negative in the third and fourth quadrants

x = 210 or x = 330 (180 + 30 and 360 − 30)
Answer: x = 210, 330

8 Exact trigonometric values

Work out the exact value of tan 45° + tan 30°
Give your answer in its simplest form, as a fraction or surd. Do not use a calculator.

$\tan 45° + \tan 30° = 1 + \frac{\sqrt{3}}{3}$

$= 1 + \frac{\sqrt{3}}{3}$

Answer: $\mathbf{1 + \frac{\sqrt{3}}{3}}$

9 Histograms with an unmarked scale

The histogram shows the masses of parcels delivered by a courier.
320 of them had a mass between 30 and 50 kg.
(a) How many had a mass between 20 and 30 kg?
(b) How many parcels are there in total?

Frequency is proportional to the area of a bar
30 to 50: bar is 4 squares high and 20 wide, area 80, which represents 320
So 1 unit of area represents 320 ÷ 80 = 4
(a) 20 to 30: 8 × 10 = 80 units of area, so 80 × 4 = 320
(b) Total area = 80 + 40 + 80 + 80 = 280, so total = 280 × 4 = 1120
Answer: (a) 320 (b) 1120

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10 Bounds of speed, density and pressure

A train travels a distance of 100 km, correct to 1 significant figure.
The journey takes 4 hours, correct to the nearest whole number.
Work out the lower bound of the speed.
Give your answer correct to 3 significant figures. (Calculator)

$50 \leq$ distance < 150; $3.5 \leq$ time < 4.5
Lower bound of speed = smallest distance ÷ biggest time = 50 ÷ 4.5
= 11.1111... = 11.1 (3 s.f.)
Answer: 11.1 km/h